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Temperature-dependent pseudogap-like features in tunnel spectra of high-T_c cuprates as a manifestation of charge-density waves

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Abstract
Temperature, T, variations of the tunnel conductance $G(V)$ were calculated for junctions between a normal metal and a spatially inhomogeneous superconductor with a dielectric gap on the nested sections of the Fermi surface or between two such superconductors. The dielectric gapping was considered to be a consequence of the charge density wave (CDW) appearance due to the electron–phonon (for a Peierls insulator) or a Coulomb (for an excitonic insulator) interactions. Spatial averaging was carried out over random domains with varying parameters of the CDW superconductor (CDWS). The calculated tunnel spectra demonstrate a smooth transformation from asymmetric patterns with a pronounced dip–hump structure at low T into those with a pseudogap depletion of the electron densities of states at higher T in the vicinity or above the actual critical temperatures of the superconducting transition for any of the CDWS domains. Thus, it is demonstrated that both the dip-hump structure and pseudogapping are manifestations of the same phenomenon. A possible CDW-induced asymmetry of the background contribution to $G(V)$ is also touched upon. The results explain the peculiar features of $G(V)$ for $Bi_2Sr_2CaCu_2O_{8+\delta}$ and other related high-T_c cuprates.

1. Introduction

The problem of the so-called pseudogap manifestations constitutes one of the major puzzles in the physics of high-T_c oxides [1–7]. In particular, noticeable deviations from the normal state behavior are observed in the temperature, T, dependence of the resistivity $\rho(T)$ far above the critical temperature T_c, reflecting changes in the electron density of states (DOS). Depletion of the DOS, different from that directly linked to superconducting pairing, has been also observed in the angle-resolved photoemission spectra (ARPES) and tunnel measurements. It should be noted that the majority of recent measurements demonstrate that, notwithstanding the apparently cumulative action of the mechanisms associated with the superconducting gap and the pseudogap on the DOS, the corresponding contributions can be distinguished experimentally, thus testifying that those phenomena are most likely competing ones of different origins. This fact was recognized some time ago for the compound $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BSCCO), which can be considered as a testing ground for studying the interplay between both kinds of gaps in question (see, e.g., tunnel, ARPES, Raman, and thermodynamic data in [8–13]). An interesting examination has been made to elucidate relationships between pseudogapping and superconductivity in the $Bi_2Sr_{2-x}R_xCuO_y$ family (R = La, Eu) [14]. Namely, ARPES spectra for both types of single crystals were obtained; it turned out that the transition from R = La to R = Eu is accompanied by a substantial reduction of T_c, whereas the pseudogap emergence temperature is *enhanced* for all charge carrier concentrations.

Moreover, ellipsometric measurements for $RBa_2Cu_3O_{7-\delta}$ (R being Y, Nd, or La) demonstrated a clear-cut difference between two energy gaps appropriate to those materials [15]. These authors also assert that the larger gap does not share the same electronic states with the smaller one. We note that it is true only to a certain extent, since the larger gap includes a contribution from the superconducting order parameter [10] (see equations below).

Recent tunnel studies of electron-doped oxides $Nd_{1.85}Ce_{0.15}CuO_{4-y}$ and $Pr_{1-x}LaCe_xCuO_{4-y}$ in high magnetic fields demonstrated that pseudogaps coexist with superconducting gaps in those compounds in the same manner as in hole-doped cuprates [16].

Spatial correlations observed between superconducting gaps and pseudogaps in the non-homogeneous BSCCO samples [17, 18] led the authors to the opposite conclusion, that the pseudogap phenomenon should have a superconducting origin. At the same time, the pseudogap versus superconducting gap competition scenario has been confirmed recently in photoemission studies for the cuprates $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_8$ [12] and $(Bi,Pb)_2(Sr,La)_2CuO_{6+\delta}$ with a low $T_c \approx 35$ K [19]. Specifically, it was shown that the momentum and T dependences for two gaps in this oxide are so unlike that they surely represent two dissimilar phenomena having different energy scales. Analogous and even more convincing results were subsequently obtained for BSCCO [20]. It is important that pseudogaps manifest themselves not only for underdoped but also for overdoped samples. For instance, tunnel measurements of BSCCO mesa structures with doping level $p = 0.19$ and $T_c = 88.3$ K revealed the existence of DOS depletion up to $T \approx 210$ K [13].

One should note that pseudogap-driven DOS depletion was observed [21] by scanning tunnel microscopy (STM) both in $La_{1.48}Nd_{0.4}Sr_{0.12}CuO_4$, which revealed so-called static stripes, and $Bi_2Sr_{1.6}Gd_{0.4}CuO_{6+\delta}$, where such stripes have not been found. This might mean that the stripe phenomena in $Bi_2Sr_{1.6}Gd_{0.4}CuO_{6+\delta}$ do exist but they are weak, or that pseudogaps are not directly connected to stripes (or any other kind of phase separation). The answer is still lacking. Broadly speaking, it is hard to define a strict boundary between 'stripe' and 'CDW' phenomena, the former being not only site-centered but bond-centered as well [22].

It is remarkable that the pseudogap appears in the voltage, V, dependences of the tunnel conductance $G(V) = \mathrm{d}J/\mathrm{d}V$, where J is the tunnel current through the junction, below T_c as well, so that one sometimes sees clear-cut coherent peaks of the superconducting origin 'inside' a wider minimum of $G(V)$ in this T-interval [8, 23]. The pseudogap features are not easy to identify, and the important problem about their universality remains unresolved. Namely, it is not clear, whether they are appropriate to all high-T_c cuprates and to all degrees of doping, including overdoped compositions [3–5, 18, 24, 25]. Therefore, taking into account that superconducting gap and pseudogap features (i) appear in ARPES [19] and tunnel [26] spectra for different momentum regions, (ii) respond differently to the external magnetic fields H [8], and (iii) change with doping in dissimilar ways [27], it is natural to draw a conclusion that the pseudogap is induced by another microscopic mechanism, differing from the more familiar Cooper-pairing one [10, 28–32].

It is worthwhile noting that the universality of pseudogaps for cuprates is not unanimously recognized. For instance, ARPES spectra found for $La_{2-x}Sr_xCuO_4$ [33] are interpreted by corresponding authors as evidence of a d-wave superconducting gap with no traces of any pseudogap.

Concerning the nature of the pseudogap, such phenomena as charge-density waves (CDWs) or spin-density waves (SDWs) [10, 31, 34, 35] (the former originating either from the availability of congruent Fermi surface segments [36, 37] or from cooperative Jahn–Teller-type instabilities [38, 39]), as well as Van Hove DOS singularities [40, 41], can be regarded responsible for it. Nevertheless, in apparent agreement with other experimental data [17, 18], there exists an alternative point of view, according to which the pseudogap may be attributed to superconducting fluctuations or pre-formed pairs emerging above T_c for certain reasons, the latter varying from model to model (see discussions in reviews [3, 42–44]). Considerations of this kind are deeply connected to the attempts to go beyond the Bardeen–Cooper–Schrieffer (BCS) picture of superconductivity, whatever its symmetry. Of course, only the totality of experimental data, which are still insufficient now, can help us to choose between various scenarios.

In addition to more or less general pseudogap phenomena, other—more specific and less pronounced—kinds of peculiarities have been observed in cuprates and especially in BSCCO. In particular, conspicuous dip–hump structures were observed in current–voltage characteristics (CVCs) using various kinds of tunnel measurements [5, 13, 45–50]. It is notable that in the S–I–N set-up, where S, I, and N stand for a high-T_c superconductor, an insulator, and a normal metal, respectively, a dip–hump structure might appear for either one bias voltage V polarity only [49] or both [50, 51], depending on the specific sample. In S–I–S symmetric junctions, dip–hump structures are observable (or not) at both CVC polarities simultaneously [49], which seems quite natural. It is also remarkable that, although the CVC for every in series of S–I–N junctions with BSCCO as superconducting electrodes was non-symmetric, especially due to the presence of the dip–hump structure, the CVC obtained by averaging over an ensemble of such junctions turned out almost symmetric, or at least its non-symmetricity turned out much lower than the non-symmetricity of each CVC taken into consideration [51].

Dip–hump structures have also been found while measuring ARPES spectra in the antinodal regions of the momentum space (i.e. in regions where the d-wave superconducting order parameter reaches its maximal values) for various high-T_c oxides (see, e.g., [52, 53]).

A common interpretation of those observations is also lacking. Nevertheless, all explanations for the dip–hump structures in the CVC of any kind of junction involving high-T_c oxides can be roughly classified into two groups (leaving aside more exotic non-Fermi-liquid scenarios [54]): (i) the dip–hump structure is considered as a by-product induced by strong-coupling effects of superconductivity [52, 53, 55] (for instance, an electron–boson interaction feature associated with a resonance mode [56]), or (ii) it is interpreted as a completely unrelated phenomenon coexisting with Cooper pairing below T_c [10]. Thus, the interpretation of the supposed origins of the dip–hump structure is almost the same as that of the pseudogap ones.

Broadly speaking, the pseudogap and the dip–hump structure have much in common. In particular, they can coexist with superconducting coherent peaks, their appearance in CVCs is to some extent random, and their shapes are sample-dependent. Therefore, a suggestion inevitably arises that those two phenomena might be governed by the same mechanism. Our main assumption is that both the pseudogap and the dip–hump structure are driven by CDW instabilities and that their varying appearances are coupled with the intrinsic, randomly inhomogeneous electronic structure of cuprates [18, 43, 57–64]. In the strict sense, according to the adopted scenario, both the dip–hump structure and the pseudogap are the manifestations of the *same* dielectric DOS depletion, the former being a result of superimposed CDW- and SC-induced CVC features below T_c. Direct spatial correlations between irregular patterns of CDW three-dimensional supermodulations [65] and topographic maps of the superconducting gap amplitudes on the BSCCO surface were recently discovered by tunnel spectroscopy [66]. We note that the very idea of competition between CDWs and superconductivity in perovskite oxides has a long history [30, 34, 67–69] and goes back to studies of $BaPb_{1-x}Bi_xO_3$ with $T_c \approx 13$ K [70, 71]. To further support the correctness of our viewpoint, we must analyze each of its aspects at a greater length.

A detailed description of our approaches to the problems of superconductors with CDWs (CDWSs), tunneling through junctions with CDWSs as electrodes, and the emergence of dip–hump structure in the CVCs of high-temperature oxides can be found elsewhere [10, 31, 72–74]. Here, we shall point out only newly obtained results and, briefly, those basic issues that are necessary for the paper to be self-contained.

2. Theory

It is noteworthy that two kinds of asymmetric behavior coexist in various cuprates. In addition to the dip–hump structure, which is the subject of the current research, and a frequently observed difference between the heights of the coherent superconducting peaks near the gap-edges [18, 24, 51, 61, 75, 76], the overall $G(V)$ dependence has an asymmetric contribution at voltages far beyond the gap region [18, 24, 25, 46, 61, 75–80]. This contribution either survives at high T for BSCCO [18, 46] or flattens out for $La_{2-x}Sr_xCuO_4$ [25]. Now, we cannot make a sound conclusion whether the asymmetric CVC background for each compound is or is not linked to the asymmetric dip–hump structures. Hence, we shall put the main emphasis on the pseudogap-like phenomena *per se*, although the interesting problem concerning asymmetric tunneling across junctions with normal (CDW gapped in our case!) electrodes [81] will also be dwelt upon. At the same time, other possibilities might occur. For instance, the $G(V)$ asymmetry might originate from the energy-dependent superconducting gap function [82] or more complicated many-body correlations violating the electron–hole equivalence [54, 83, 84]. On the other hand, an asymmetric behavior—but only in the case of anisotropic d-wave pairing—was proposed to be a consequence of an interaction between a superconducting electrode and a non-superconducting substrate [85]. The mutual shortcoming of those interpretations consist in the inevitability of the CVC asymmetry, whereas the character and the very existence of the latter is highly unstable for a certain given material.

Alternatively, we consider the problem by treating high-T_c oxides as CDWSs in the framework of the theory first formulated by Bilbro and McMillan [86] for coexisting s-wave superconductivity and CDWs and, later, developed by us in a closed self-consistent form [72]. This theory assumes nesting conditions to be fulfilled only for certain ($i = 1, 2$) Fermi surface (FS) sections (d), which can be associated with a dielectric order parameter and become dielectrically gapped below some temperature $T_d > T_c$, while the rest of the FS ($i = 3$) remains ungapped (n) down to T_c, so that the dielectric CDW gapping is only partial. The degree of such a FS separation is described by a parameter ($0 < \mu < 1$)

$$\mu = \frac{N_d(0)}{N(0)} \equiv \frac{N_d(0)}{N_d(0) + N_n(0)}, \tag{1}$$

where $N(0)$ is the total quasiparticle DOS on the ungapped FS (above T_d) and $N_d(0)$ and $N_n(0)$ are the quasiparticle DOSs at the d and n FS sections, respectively. On the other hand, the so-called strong mixing of the four-particle interaction results in the extension of the isotropic superconducting order parameter over the whole FS [10, 86]. The Bilbro–McMillan partitioning of the FS with respect to dielectric gap formation has been recently confirmed for BSCCO by photoemission spectroscopy [20]. It was clearly shown that the 'pseudogapless' Fermi arc region located near the nodal momentum space direction elongates with doping, i.e. in our terms the parameter μ grows. As for the superconducting gap, it possesses a canonical $d_{x^2-y^2}$-form only for $T \ll T_c$, whereas in the transition temperature range $T < T_c$ the momentum dependence of the superconducting order parameter is more complex. Raman studies also confirmed the interplay between the superconducting gap and pseudogap in the antinodal region for another oxide, $HgBaCuO_{4+\delta}$ [87].

The deviations of the superconducting order parameter from the pure d-wave behavior (see, e.g., the discussion of electron-doped cuprates [88]) or/and the influence of the dielectric order parameter could be responsible for this phenomenon.

The mean-field Hamiltonian of the CDWS includes the interaction terms responsible for the dielectric and superconducting gaps of the FS. If there had been no CDW gap, the 'parent' superconductor would have been characterized by a perfect BCS behavior with a 'parent' zero-T superconducting order parameter Δ_0^* and a 'bare' $T_c^* = \frac{\gamma}{\pi}\Delta_0^*$ ($\gamma = 1.7810\cdots$ is the Euler constant, and the Boltzmann constant $k_B = 1$). In the other extreme case, where the Cooper pairing is absent, only a parent partially gapped CDW-metal (CDWM) phase with the dielectric order parameter $\tilde{\Sigma}_0^* = \Sigma_0^* e^{i\varphi}$ and the CDW transition temperature $T_d^* = \frac{\gamma}{\pi}\Sigma_0^*$ would have developed.

It can be shown that in the framework of our model for a homogeneous CDWS the superconducting and dielectric gaps may coexist only if $\Sigma_0^* > \Delta_0^*$, so that $T_d^* > T_c^*$. But since quasiparticles are engaged both in superconducting and dielectric gapping, simultaneous account of both brings about a further reduction of the actual T_c with respect to the actual $T_d = T_d^*$. Within the interval $T_c < T < T_d$, the actual CDWS is identical to the 'parent' CDWM with the dielectric order parameter

$$\Sigma(T) = \Sigma_0^* \mathrm{M\ddot{u}}(T/T_d), \tag{2}$$

where $\mathrm{M\ddot{u}}(x)$ is the normalized Mühlschlegel dependence with $\mathrm{M\ddot{u}}(0) = 1$ and $\mathrm{M\ddot{u}}(1) = 0$, and the gap

$$D(T) = |\Sigma(T)| \tag{3}$$

on the d FS sections. (We recall that the Mühlschlegel dependence $\mathrm{M\ddot{u}}(x)$ is the solution of the classical s-wave BCS–Mühlschlegel gap equation [89, 90].) In this temperature interval, the gap $D(T)$ has a pure dielectric origin and is governed by the dielectric order parameter Σ_0^* only. Within the lower T range $0 \leqslant T \leqslant T_c$, the actual CDWS is gapped both on the n and d FS sections, but differently. On the n section, there appears a superconducting gap

$$\Delta(T) = \Delta_0 \mathrm{M\ddot{u}}(T/T_c), \tag{4}$$

where

$$\Delta_0 = (\Delta_0^* \Sigma_0^{*-\mu})^{\frac{1}{1-\mu}} \tag{5}$$

and $T_c = \frac{\gamma}{\pi}\Delta_0$, so that the gapping is BCS-like with Δ_0 and T_c renormalized relative to their 'bare' values Δ_0^* and T_c^*, respectively. Equation (5) represents a suppression of superconductivity by dielectric gapping. It is remarkable that weak local correlations between the observed gap Δ magnitude and the nano-structure of the BSCCO surface were registered by measuring the surface map of the work function [91]. On the d sections, the gap

$$D(T) = \left(\Delta^2(T) + \Sigma^2(T)\right)^{1/2} \tag{6}$$

appears, for example, in accordance with experimental observations for $HgBaCuO_{4+\delta}$ [87]. It is of interest that although the gap D is a 'combined' one at $T < T_c$ its temperature dependence in this range is identical to that of the 'purely dielectric' gap at $T > T_c$ and is described by the same formula (3).

We should emphasize different roles of the phases of the order parameters. Concerning the superconducting order parameter, its phase may be arbitrary unless we are interested in the Josephson current across the junction, as for conventional BCS superconductors [92]. In any case, the superconducting order parameter phase does not influence the thermodynamic properties of the CDWS [93]. On the other hand, the dielectric order parameter phase φ also does not affect the *thermodynamic* properties of CDWSs [72] but governs quasiparticle CVCs of junctions with a CDWS as an electrode [28, 94]. The value of φ can be pinned by various mechanisms in both excitonic and Peierls insulators, so that φ acquires a value of either 0 or π in the first case [95] or is arbitrary in the unpinned state of the Peierls insulator [36]. At the same time, in the case of an inhomogeneous CDWS, which will be discussed below, a situation can be realized where φ values are not correlated over the junction area. Then, the contributions of elementary tunnel currents may compensate one another to some extent, and this configuration can be phenomenologically described by introducing a certain effective phase φ_{eff} of the dielectric order parameter. If the spread of the phase φ is random, the most probable value for φ_{eff} is zero, and the CVC for a non-symmetric junction involving CDWS becomes symmetric.

On the other hand, the inverse situation may occur, where the nominally 'symmetric' tunnel junction between thermodynamically identical CDWSs would be characterized by a certain imbalance of effective φ_{eff} values between electrodes. For such a case, the theory predicts, as a variant, that more or less non-symmetric CVCs could be observed [94], as happened sometimes in experiments.

Most often, CVCs for cuprate–I–N (i.e. S–I–N) junctions reveal the dip–hump structure only at $V = V_S - V_N < 0$ [45–47], so that the occupied CDWS electron states below the Fermi level are probed. In our approach, it corresponds to the phase φ close to π. This preference may be associated with some unidentified features of the CDW behavior near the sample surface. The true explanation of this fact can be given only at the microscopic level, which is beyond the scope of our study. Nevertheless, one should pay attention to the fact that the value $\varphi = \pi$ in the vicinity of the junction corresponds to the maximum of the majority carrier depletion at the surface. Those carriers—in this case, holes—in the $La_{2-x}Sr_xCuO_4$ and BSCCO oxides are supplied to a neighboring Cu–O superconducting layer by a La–O or Sr–O one, respectively. Near the sample surface, itinerant charges are more confined to the donor La–O or Sr–O layer, which reduces their electrostatic energy. This basic idea has been used to construct a specific model in which an ideal symmetric CVC with a bulk hole density is transformed into an asymmetric one modified by the depletion layer [96]. As a consequence, the coherent peaks become unequal and the $G(V)$ curve as a whole becomes warped. These considerations show that all asymmetric CVC features might be related to one another.

On the other hand, there are S–I–N junctions where dip–hump structures are similar for both V polarities [48–51]. As for those pseudogap features, which were unequivocally observed mostly at high T, no preferable V-sign of their manifestations was found. We note that the symmetricity of $G(V)$ might be due either to the microscopic advantage of the CDW state with $\varphi = \pi/2$ or to the superposition of different current paths in every measurement covering a spot with a linear size of a CDW coherence length at least.

Both possibilities should be kept in mind. The variety of $G(V)$ patterns in the S–I–N set-up for the same material and with identical doping is very remarkable, showing that the tunnel current is rather sensitive to the dielectric order parameter phase φ. At the same time, if one scans the surface of an inhomogeneous BSCCO sample by the scanning tunnel microscopy (STM) method, it becomes possible to distinguish a much more subtle feature, namely a modulation of $|\Delta|$ of about 5% [66]. The origin of the superconducting order parameter amplitude variation (the so-called pair density wave [97]) is not clear [98] and is, probably, intimately related to the superconductivity mechanism itself, being beyond the scope of our approach.

Nevertheless, the very appearance of the superconducting domain structure for cuprates with local domain-dependent gaps and critical temperatures [99] seems quite plausible for materials with small coherence lengths. Essentially the same approach was proposed earlier to explain the superconducting properties of magnesium diboride [100].

3. Evidence of gap spread

Randomness is very important in this context, since it is very perceptible and leads to the existence of a very broad spectrum of energy gaps observed by the STM technique, e.g. on the surface of BSCCO and other cuprate samples. The actual spread of the gap values is so large that the overlap of the pseudogap and superconducting gap distributions in this spectrum has been overlooked so far. We suppose that the smaller energy gap with high and narrow coherence peaks in the CVC is of superconducting origin (see equation (4)), while the larger gap corresponding to lower and broadened peaks (the spread of these peaks on one or both CVC branches may be so large that they can be effectively smoothed out) is of 'mostly CDW' origin (see equation (6)). It is our second basic idea, which, together with the assumption of the CDW normal state background, comprises the basis of our concept.

Indeed, STM measurements of BSCCO revealed wide random distributions of apparent gaps for various doping levels (hole concentration of 0.08–0.22) and T [17, 18, 80, 101]. The same technique applied to $Bi_{2-x}Pb_xSr_2CuO_y$ oxides demonstrated random scatter of gaps, with a presumably superconducting order parameter Δ ranging from 13 to 30 meV over the 12.5×12.5 nm^2 area [62]. At the same time, the dependence $G(V)$ measured for some areas exhibited a pseudogap-like behavior with the pseudogap feature $\Pi \leqslant$ 30 meV (D in our interpretation). A similar picture was observed for almost optimally doped BSCCO specimens [63]: STM spectra revealed 30 Å spots with 'low' (25–30 meV) and 'high' (50–75 meV) values of the gaps. In a trilayer material $TlBa_2Ca_2Cu_2O_{10-\delta}$, a wide spread of apparent Δ from 12 to 71 meV exists [50]. A similar spread was found for a multilayered high-T_c cuprate $(Cu, C)Ba_2Ca_3Cu_4O_{12+\delta}$ with $T_c = 117$ K [64]. The gap histograms plotted for $Bi_2Sr_{1.6}La_{0.4}CuO_{6+\delta}$ and, especially, $Bi_2Sr_{1.6}Gd_{0.4}CuO_{6+\delta}$ have a clear-cut two-gap shape (see figure 3 in [102]). STM studies of lightly hole-doped cuprates $Ca_{1.88}Na_{0.12}CuO_2Cl_2$ and $Bi_2Sr_2Dy_{0.2}Ca_{0.8}Cu_2O_{8+\delta}$ demonstrated that the observed randomness has an electron glass character with chaotically dispersed $4a_0$-wide unidirectional electronic domains [24], the structure qualitatively described by unidirectional bond-centered CDWs [22].

So far there has only been indirect evidence for the probably non-homogeneous phase separation in $La_{2-x}Sr_xCuO_4$ revealed by the magnetic susceptibility measurements of corresponding single crystals with varying x [103]. Namely, it was shown that the Meissner volume fraction increases with x for both underdoped and overdoped compositions. Such a gradual behavior of the principal superconducting characteristic agrees with the conclusions drawn from neutron scattering experiments that $La_{2-x}Sr_xCuO_4$, contrary to the statically ordered Nd-doped compound $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$, is in a nearly ordered (dynamically ordered) stripe liquid phase [59, 60, 104, 105] or in a percolative state with superconducting clusters of negative-U centers responsible for superconductivity [106, 107]. Recently, direct observations of spatially inhomogeneous gap distributions in $La_{2-x}Sr_xCuO_4$ were carried out by STM [108]. In addition, the authors of [108] found unusual kinks of the tunnel conductivity at energies ± 5 meV, i.e. *inside* the superconducting energy gap.

Unfortunately, the microscopic nature of random non-homogeneities in BSCCO and other hole-doped cuprates remains obscure, although some correlations between various quantities have been noticed [43, 61] and some reasonable preliminary conjectures have been stated [109–111]. It seems that the statement made in [18] on the basis of the steadiness of the ratio $2\Delta_{loc}/T_p \approx 8$ (here, Δ_{loc} is a chosen local representative of the observed gap distribution and T_p is the corresponding onset temperature) that local pairing in non-homogeneous BSCCO samples is the same above and below T_c is at least premature. Such 'non-BCS' values are typical of the superconducting order parameters of cuprates [112] as well as the dielectric order parameters of CDW partially gapped metals and insulators [113]. In the STM approach of [18], it was impossible to distinguish between two kinds of gaps.

It is worth stressing once more that random patches with different electronic properties include domains with regularly modulated charge distributions. This concerns BSCCO and similar materials, for which stripe-like and checkerboard-like structures are revealed [5, 43, 59, 60, 75, 114, 115]. In this paper, they are interpreted as CDW manifestations, although elucidation of the role and determination of the degree of nesting in various cuprates demands further study.

4. Current–voltage characteristics

Based on the reasons stated above and using the previously developed approach [10, 28, 73], we calculated quasiparticle tunnel CVCs $J(V)$ for two typical experimental set-ups. Namely, we considered CDWS–I–N and CDWS–I–CDWS junctions. The Green function method follows the classical approach of Larkin and Ovchinnikov [116].

In the most general case (the CDWS–I–CDWS′ junction), there are nine components of the quasiparticle tunnel current

$$J(V) = \sum_{i,i'=n,d,c} J_{ii'}(V), \tag{7}$$

each of the following structure:

$$J_{ii'} \propto \frac{1}{R}\,\mathrm{Re}\int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\omega\, \frac{\mathrm{Im}\ G_{i'}(\omega')G_i(\omega)}{\omega' - \omega + eV + i0}. \quad (8)$$

The difference $V = V_{\mathrm{CDWS}'} - V_{\mathrm{CDWS}}$ is the bias voltage across the junction reckoned from the potential of the CDWS electrode. Here R is the tunnel resistance of the junction in the normal state

$$R^{-1} = 4\pi e^2 N^{\mathrm{CDWS}}(0) N^{\mathrm{CDWS}'}(0) \left\langle |\mathsf{T}|^2 \right\rangle_{\mathrm{FS}}. \quad (9)$$

The parameter $e > 0$ is the elementary charge. The quantity $\langle |\mathsf{T}|^2\rangle_{\mathrm{FS}}$ in equation (9) mean averaging of the square of the, generally speaking, momentum-dependent tunnel matrix elements $\mathsf{T}_{\mathbf{pq}}$ over the electrode FSs. The conventional assumption [117] of the constant $\langle |\mathsf{T}|^2\rangle_{\mathrm{FS}}$ corresponds to the so-called incoherent tunneling. On the other hand, if the wavevector components parallel to the junction plane are at least partially preserved, a unique R no longer exists, so that the corresponding equation for the tunnel current differs from equation (8) and includes two extra integrals over the FS (see, e.g., [56, 118]). A thorough analysis [119, 120] shows that, in BSCCO, tunneling is very close to incoherent, so that we restrict ourselves to approximation (9).

According to equation (8), each component of the tunnel current is a functional of the product of two temporal Green functions, one for each CDWS electrode. A CDWS is characterized by three Green functions: $G_d^{\mathrm{CDWS}}(\omega)$, $G_n^{\mathrm{CDWS}}(\omega)$, and $G_c^{\mathrm{CDWS}}(\omega)$ [10, 28, 73]. The first two describe quasiparticles from the dielectrically gapped and ungapped FS sections, whereas the last one is proportional to the dielectric order parameter and represents the electron–hole pairing.

On the other hand, the normal metal is characterized by a single Green function $G^{\mathrm{N}}(\omega)$. Consequently, the series for the current $J^{\mathrm{ns}}(V)$ through a for non-symmetric CDWS–I–N junction includes only three components J_i^{ns}:

$$J^{\mathrm{ns}}(V) = \sum_{i=n,d,c} J_i^{\mathrm{ns}}(V), \quad (10)$$

with corresponding modifications in expressions (8) and (9).

The functions $G_i^{\mathrm{CDWS}}(\omega)$ and $G^{\mathrm{N}}(\omega)$ can be obtained from the corresponding temperature Green functions of CDWS and normal metal, respectively, by the well-known procedure of [73, 116]. In the CDWS–I–N case, the explicit expressions for the components of the tunnel current across the junction are

$$J_n^{\mathrm{ns}} = \frac{(1-\mu)}{4eR}\int_{-\infty}^{\infty} d\omega\, K(\omega, V, T)|\omega| f(\omega, \Delta), \quad (11)$$

$$J_d^{\mathrm{ns}} = \frac{\mu}{4eR}\int_{-\infty}^{\infty} d\omega\, K(\omega, V, T)|\omega| f(\omega, D), \quad (12)$$

and

$$J_c^{\mathrm{ns}} = \frac{\mu\Sigma\cos\varphi}{4eR}\int_{-\infty}^{\infty} d\omega\, K(\omega, V, T)\ \mathrm{sgn}(\omega)\ f(\omega, D), \quad (13)$$

where

$$K(\omega, V, T) = \tanh\frac{\omega}{2T} - \tanh\frac{\omega - eV}{2T} \quad (14)$$

and

$$f(\omega, x) = \frac{\theta(|\omega| - x)}{\sqrt{\omega^2 - x^2}}. \quad (15)$$

Hereafter, the Boltzmann constant $k_{\mathrm{B}} = 1$. One sees that the dielectric order parameter phase φ explicitly appears in term (13). It is due to the electron–hole pairing, and hence this component of the *quasiparticle* current is to some extent an analogue of the *Josephson* current between superconductors.

In another important particular case, namely a symmetric CDWS–I–CDWS junction, sum (7) for the tunnel current is reduced to five different summands, the structures of which are similar to that of equation (8):

$$J^{\mathrm{s}}(V) = \sum_{i=nn,dd,cc,dn,nd} J_i^{\mathrm{s}}(V). \quad (16)$$

After straightforward calculations, they read

$$J_{nn}^{\mathrm{s}} = \frac{(1-\mu)^2}{4eR}\int_{-\infty}^{\infty} d\omega\, K(\omega, V, T)|\omega| f(\omega, \Delta) \times |\omega - eV| f(\omega - eV, \Delta), \quad (17)$$

$$J_{dd}^{\mathrm{s}} = \frac{\mu^2}{4eR}\int_{-\infty}^{\infty} d\omega\, K(\omega, V, T)|\omega| f(\omega, D) \times |\omega - eV| f(\omega - eV, D), \quad (18)$$

$$J_{cc}^{\mathrm{s}} = \frac{\mu^2\Sigma^2\cos^2\varphi}{4eR}\int_{-\infty}^{\infty} d\omega\, K(\omega, V, T)\,\mathrm{sgn}(\omega) f(\omega, D) \times\ \mathrm{sgn}(\omega - eV) f(\omega - eV, D), \quad (19)$$

$$J_{dn}^{\mathrm{s}} = J_{nd}^{\mathrm{s}} = \frac{\mu(1-\mu)}{4eR}\int_{-\infty}^{\infty} d\omega\, K(\omega, V, T)|\omega| f(\omega, D) \times |\omega - eV| f(\omega - eV, \Delta). \quad (20)$$

Equations (10)–(20) form the basis for calculations. Strictly speaking, they must be supplemented with a proper account of the non-homogeneous background inevitably existing in many cuprates, including BSCCO. In this connection, our theory assumes the combination CDW+ inhomogeneity to be responsible for the appearance of the dip–hump structure, as expounded in [74]. The main conclusion is that it is the dispersion of the parameter Σ_0^*—and, as a result, the D-peak smearing (the Δ-peak also becomes smeared but to a much lesser extent)—that gives the dominant contribution to the effect. The value of the degree of FS gapping μ is mainly responsible for the amplitude of the dip–hump structure. At the same time, neither the scattering of the parameter μ nor that of the superconducting order parameter Δ_0^* can result in the emergence of a smooth dip–hump structure, so that sharp CDW features remain unaltered. Therefore, for our purpose it was sufficient to average only over Σ_0^* rather than simultaneously over all CDWS parameters, although the variation of any individual parameter or the variations of all of them simultaneously made the resulting theoretical CVCs more similar to experimental ones.

The parameter Σ_0^* was regarded as distributed within the interval $[\Sigma_0^* - \delta\Sigma_0^*, \Sigma_0^* + \delta\Sigma_0^*]$. The normalized weight function $W(x)$ was considered as a bell-shaped fourth-order polynomial within this interval and equal to zero beyond it (see discussion in [74]). In any case, the specific form of $W(x)$ is not crucial for our final results and conclusions.

Our approach is, in essence, the BCS-like one. This means, in particular, that we do not take a possible quasiparticle 'dressing' by impurity scattering and the electron–boson interaction, as well as the feedback influence of the superconducting gapping, into account [121, 122]. Those effects, which are important *per se*, cannot qualitatively change the random two-gap character of superconductivity in cuprates.

As has already been mentioned, we assume that both Δ and Σ are s-wave order parameters. Nevertheless, our approach is applicable to superconductors with d-wave symmetry, which is usually considered true for at least hole-doped cuprates [123]. The technical distinction lies in the dependence of the superconducting order parameter on the angle θ in the Cu–O plane, which leads to the additional mathematical complications [124], although the physical picture remains essentially the same. In any case, the intra-gap (we mean the superconducting gap here) region is of minor importance in view of the problem studied in our paper.

For isotropic superconductors, to which the adopted phenomenological model is unequivocally applicable, the CVC patterns obtained below have not yet been observed, mainly because of the experimental difficulties in finding the regions of coexistence in the respective phase diagrams. Nevertheless, recently the old idea of the incomplete mutual destruction between both kinds of pairings [86, 125–128] has been recently revived, e.g. for sulfur [129], $Y_5Ir_4Si_{10}$ [130], and Cu_xTiSe_2 [131].

5. Averaging or differentiation first?

In the case of inhomogeneous electrodes, the spread δx of each of the electrode parameters $x = (\Delta_0, \Sigma_0, \mu)$ results in a smearing, to a certain extent, of the gap-driven singularities. Every $J(V)$-point is actually an average of weighted contributions from different neighboring patches. If we are interested in differential CVCs, $G(V)$, the following methodological consideration has to be of importance for junctions, where both electrodes possess energy gaps of whatever nature in their electron spectra, the S–I–S junction being a particular case.

Consider a junction between two inhomogeneous electrodes with BCS-like gaps (let the gaps in either electrode be scattered around the values Δ_1 and Δ_2), and let us analyze $J(V)$ and $G(V)$, in the vicinity of the bias voltage $V = (\Delta_1 + \Delta_2)/e$ (see figure 1, the same is valid for the negative-bias branch). In the homogeneous case, the $J(V)$ dependence would look like a dashed curve in panel (a), namely there would be a finite current jump here, and the current derivatives would be finite on both sides of the jump. Therefore, the dependence $G(V)$ would look like a dashed curve in panel (b). Its step shape is induced by the difference between the $G(V)$ values to the left and to the right of the voltages $V = \pm(\Delta_1 + \Delta_2)/e$, but the magnitude of the $\Delta J(eV = \Delta_1 + \Delta_2)$ jump itself becomes *totally overlooked* under this procedure. The $G(V)$ profiles are *finite* smooth curves on both sides of the jump. Averaging $G(V)$ over inhomogeneous electrodes is reduced to its averaging over various eV positions of the jump and, correspondingly, over various $G(V)$ profiles in their vicinity. Therefore, it becomes clear that such an averaging cannot result in anything else but a smeared, distorted step at $eV \approx \Delta_1 + \Delta_2$ in the $\langle G(V)\rangle$ dependence (dotted curve in panel (b)). Hereafter, the notation $\langle\cdots\rangle$ means averaging over the distributions of all parameters, non-homogeneous at the electrode surface.

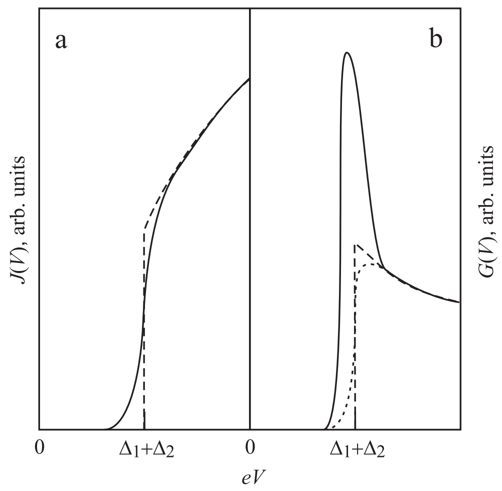

Figure 1. Schematic illustration of the peak emergence in the dependence of the differential conductance $G(V) = dJ/dV$ of a tunnel junction between two inhomogeneous gapped electrodes. Here, J is the quasiparticle current through and V the bias voltage across the junction. For other explanations see the text.

On the other hand, raw experimental data are no more than a $\langle J(V)\rangle$ dependence. Averaging over inhomogeneous electrodes gives rise to the smearing of the $\Delta J(eV = \Delta_1+\Delta_2)$ (a solid curve in panel (a)), but, although being distorted, the current jump survives averaging, so that its following differentiation would result in the emergence of a $d\langle J\rangle/dV$ peak in the vicinity of the $V \approx (\Delta_1 + \Delta_2)/e$ voltage (solid curve in panel (b)) rather than a smeared step (dotted curve). Thus, an attempt to calculate the averaged $\langle G(V)\rangle$ characteristic by averaging $G(V)$ characteristics inherent to junctions composed of homogeneous electrodes would inevitably bring about an incorrect result.

That or another method of device-assisted differentiation is reduced to the calculation of a finite difference $\delta\langle J\rangle/\delta V$ in some voltage interval δV. However, it is evident that, if the value of δV is small enough, the difference between $d\langle J\rangle/dV$ and $\delta\langle J\rangle/\delta V$ becomes negligible, taking all other circumstances (experimental errors) into account. Nevertheless, for the sake of consistency, it is desirable to follow just the experimental procedure and differentiate the averaged $\langle J(V)\rangle$ dependence numerically.

Break junctions without any doubt belong to the class of junctions considered above. Hence, while calculating differential CVCs for them, the sequence of averaging and differentiation operations is of importance. On the other hand, if one of the electrodes is a normal metal and the counter-electrode has a BCS-like gap Δ (a S–I–N junction), the derivative dJ/dV has a divergence point—an inverse square root—at $eV = \Delta$. This circumstance ensures the existence of gap-like coherent peaks in the $G(V)$ dependence

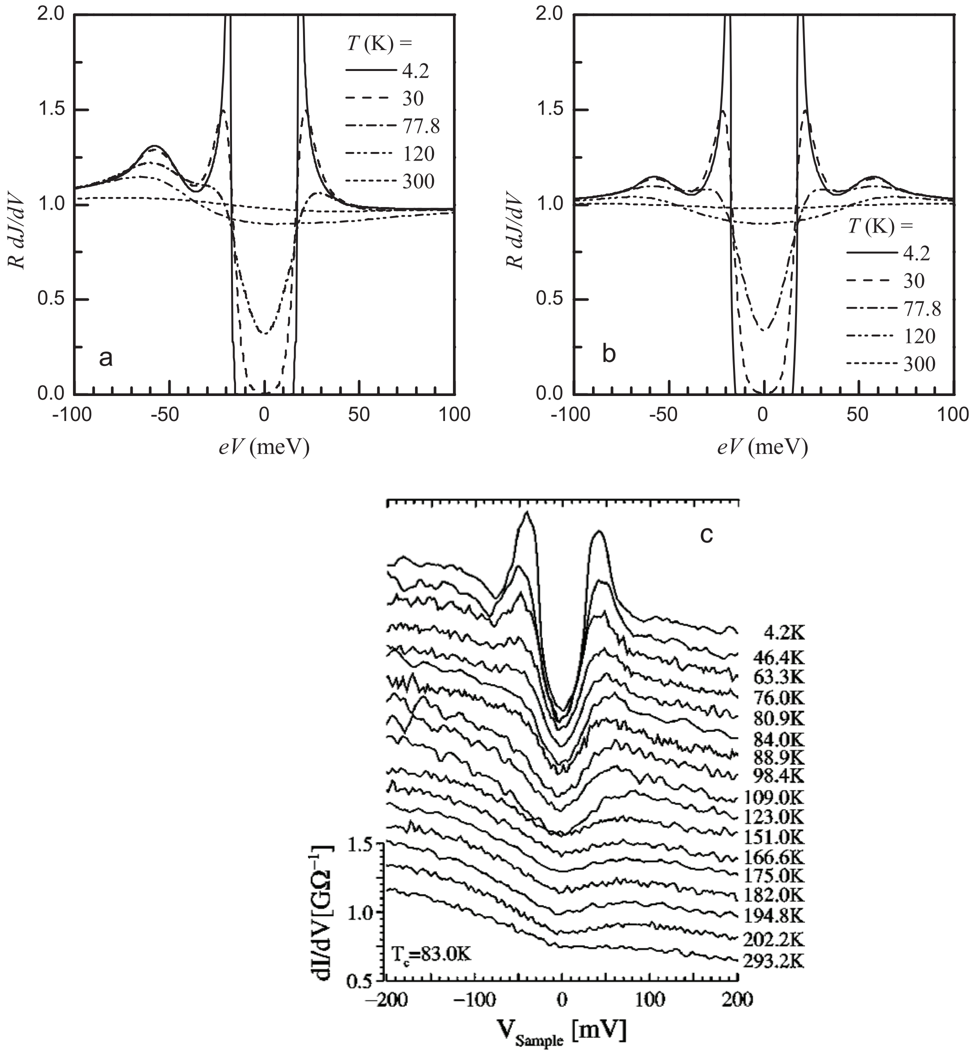

Figure 2. $G(V)$ dependences of the tunnel junction between an inhomogeneous CDWS and a normal metal for various temperatures T. The dielectric order parameter phase $\varphi = \pi$ (panel (a)) and $\pi/2$ (panel (b)), and the spread of the dielectric order parameter-amplitude $\delta\Sigma_0^* = 20$ meV. All other parameters are indicated in the text. (panel (c)) STM spectra for underdoped BSCCO–Ir junctions registered at various temperatures. Reprinted with permission from [46]. Copyright 1998 by the American Physical Society.

at $eV \approx \Delta$ for both operation sequences, although the specific parameters of the peaks, such as the amplitude and the width, are somewhat different for each case.

Hence, to obtain a theoretical differential CVC for a symmetric junction, which would reproduce an experimental one obtained by a modulation technique of some kind, one should first calculate the averaged dependence $\langle J(V)\rangle$ and then differentiate it to obtain $\mathrm{d}\langle J\rangle/\mathrm{d}V$ or calculate the numerical difference $\delta\langle J\rangle/\delta V$, the latter procedure being even closer to the experimental data treatment. We would like to emphasize once more that the whole consideration is valid for both superconductivity- and CDW-driven gaps, because their DOSs have the same structure due to the similarity between relevant coherent factors [95].

6. Results of the calculations and discussion

The results of calculations presented below show that the same CDW + inhomogeneity combination can also explain well the pseudogap phenomena at high temperatures, when the dip–hump structure is smoothed out. Thus, the goal of the calculation of tunnel CVCs is to track the details of the apparent dip–hump structure transformation into the pseudogap DOS depletion for non-symmetric and symmetric junctions, involving cuprate electrodes. We consider the CDW-driven phenomena, dip–hump structure included, as the tip of an iceberg, a huge part of which is hidden by strong superconducting manifestations, less influenced by randomness than their CDW counterpart. To uncover this part, one should raise the temperature, which is usually done with no reference to the dip–hump structure, the latter being substantially smeared by the Fermi-distribution thermal factor. It is this DOS depletion phenomenon that is connected to the pseudogapping phenomena [1, 3, 5, 18, 42, 43].

An example of the transformation of the dip–hump structure-decorated tunnel spectra into the typical pseudogap-like ones is shown in figure 2 for CDWS–I–N junctions with $\varphi = \pi$ (panel (a)) and $\pi/2$ (panel (b)). The CDWS parameters

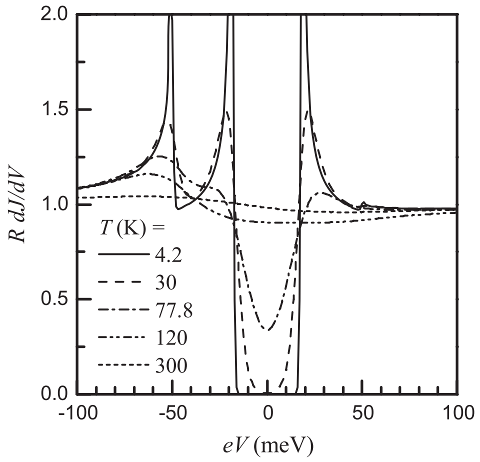

Figure 3. Temperature evolution of the dispersionless $\varphi = \pi$ curve.

are $\Delta_0^* = 20$ meV, $\Sigma_0^* = 50$ meV, $\mu = 0.1$, and $\delta\Sigma_0^* = 20$ meV; the temperature $T = 4.2$ K. For this parameter set, the 'actual' superconducting critical temperatures T_c of CDWS domains lie within the interval 114–126 K, and the temperatures of the CDW phase transition T_d is in the range 197–461 K. The value $\varphi = \pi$ was selected because this case corresponds to the availability of the dip–hump structure in the negative-voltage branch of the non-symmetric CVC, and such an arrangement is observed in the majority of experimental data. From this figure, the transformation of the pattern of the CVC including the dip–hump structure calculated for temperatures well below T_c into a pseudogap-like structure in the vicinity of T_c or above becomes lucid. As T grows and transforms the dip–hump structures into pseudogaps, all remnants of the CDW coherent peaks quickly disappear in contrast to their superconducting counterparts. The difference is due to the preliminary weakening of the CDW-related peaks at $T = 0$ by spatial averaging, so they become extremely vulnerable to thermal smearing.

The asymmetric curves displayed in panel (a) are similar to the measured STM $G^{ns}(V)$ dependences for overdoped and underdoped BSCCO compositions [18]. The overall asymmetric slope of the experimental curves, which is independent of gaps and T, constitutes the main distinction between them and our theoretical results. It might be connected to the surface charge carrier depletion induced by CDWs and mentioned above. Another interesting feature of our results is the modification of the Δ peak and the shift of its position. Although Δ diminishes as T grows, the Δ peak moves towards higher bias voltages; such a behavior of the Δ peak is undoubtedly associated with its closeness to the Σ-governed dip–hump structure. In experiments, a confusion of identifying this Δ-driven singularity with a pseudogap feature may arise, since the observed transformation of Δ features into pseudogap (D) ones looks very smooth [5].

It is notable that, in the case of asymmetric $G^{ns}(V)$, the low-T asymmetry preserves well into the normal state, although the dip–hump structure as such totally disappears. The extent of the sample randomness substantially governs CVC patterns. Therefore, pseudogap features might be less or more pronounced for the same materials and doping levels. At the same time, for the reasonable scatter of the problem parameters, the superconducting coherent peaks always survive the averaging (below T_c, of course), in accordance with experiment. Our results also demonstrate that the dependences $\Delta(T)$ taken from the tunnel data may be somewhat distorted in comparison to the true ones due to the unavoidable Δ versus Σ interplay. One should stress that, in our model, 'hump' positions, which are determined mainly by Σ rather than by Δ, anticorrelate with true superconducting gap values Δ inferred from the coherent peaks of $G(V)$. This is exactly what was found for non-homogeneous BSCCO samples [80].

Making allowance for CDWs, it might seem possible to consider the pseudogap manifestation as a temperature-smeared D peak without any Σ scatter. Indeed, as figure 3 demonstrates, the high-temperature CVC patterns in this case would be similar to those in figure 2(a). But the low-temperature ones would be substantially different in this case from what is observed experimentally. The main discrepancy is the relationship between the amplitudes of the Δ- and D-induced peaks. In all other aspects, they are rather similar, so that the true, i.e. inhomogeneity-induced, origin of the CDW-peak broadening can be easily confused with a trivial temperature smearing.

Similar CDW-related features should be observed in the CVCs measured for symmetric CDWS–I–CDWS junctions. The $G(V)$ dependences for this case with the same sets of parameters as in figure 2 are shown in figure 4. As is readily seen, the transformation of the symmetric dip–hump structure pattern into the pseudogap-like picture is similar to that for the non-symmetric junction. This simplicity is caused by a smallness of the parameter $\mu = 0.1$, so that the features at $eV = \pm 2D$, which are proportional to μ^2, are inconspicuous on a chosen scale. At the same time, two terms given by the equation (20) lead to the prominent *square-root singularities* at $eV = \pm(D + \Delta)$. Note that for *arbitrary* magnitudes of Σ and Δ, those energies do not coincide with the values $\pm(\Sigma + \Delta)$ [in more frequently used notation, $\pm(\Delta_{PG} + \Delta_{SG})$, the subscripts PG and SG denoting the pseudogap and the superconducting gap, respectively], which can sometimes be met in literature [13]. The latter relation becomes valid only for $\Sigma \gg \Delta$.

There is another reason why the singularities of $G^s(V)$ at $eV = \pm 2D$ cannot be observed. Specifically, for a CDWM–I–CDWM junction, the discontinuities of the total quasiparticle current induced by terms (18) and (18) have different signs and annihilate each other, so that only a finite jump appears in the differential conductivity conductance [132]

$$\Delta G^s_{CDW}(eV = \pm 2D) = \frac{\pi\mu^2}{2R}\tanh\left(\frac{D}{2T}\right). \qquad (21)$$

On the other hand, a conventional S_{BCS}–I–S_{BCS} structure reveals a jump

$$\Delta J^s_{BCS}(eV = \pm 2\Delta_{BCS}) = \frac{\pi\Delta_{BCS}}{2eR}\tanh\left(\frac{\Delta_{BCS}}{2T}\right) \qquad (22)$$

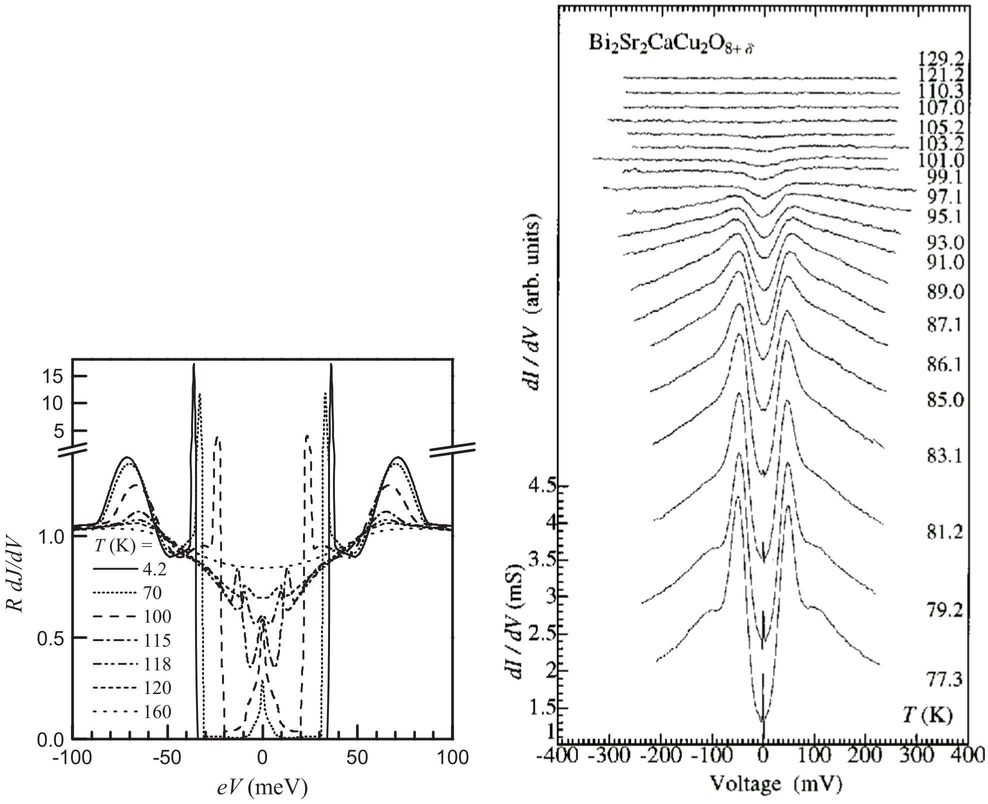

Figure 4. Left panel: the same as in figure 2(a), but for a symmetric CDWS–CDWS junction. Right panel: temperature variations of experimental differential CVCs for a BSCCO break junction. Reprinted with permission from [1]. Copyright 1999 by the American Physical Society.

of the quasiparticle *current* [133]. A similar jump with accurate notations emerges for the CDWS–I–CDWS junction at $eV = \pm 2\Delta$ [28]

$$\Delta J^{s}_{\text{CDWS}}(eV = \pm 2\Delta) = \frac{\pi\Delta(1-\mu)^2}{2eR}\tanh\left(\frac{\Delta}{2T}\right). \quad (23)$$

Therefore, for a symmetric junction with a strictly perfect homogeneous CDWS, the theory leads to a δ-function singularity in $G^{s}(V)$ at this voltage. For an infinitesimally larger V, the conductance is finite

$$G^{s}_{\text{CDWS}}[eV = \pm(2\Delta + 0)] = \frac{3\pi}{8R}. \quad (24)$$

We note that in our case of the CDWS–I–CDWS junction terms (18) and (19) will no longer compensate each other at $eV = \pm 2D$ [73]

$$\Delta J^{s}_{\text{CDWS}}(eV = \pm 2D) = \Delta J^{s}_{dd,\text{CDWS}}(eV = \pm 2D) + \Delta J^{s}_{cc,\text{CDWS}}(eV = \pm 2D)$$
$$= \frac{\pi\mu^2\Delta^2}{2RD}\tanh\left(\frac{D}{2T}\right). \quad (25)$$

According to equations (25) and (23),

$$\frac{\Delta J^{s}_{\text{CDWS}}(eV = \pm 2D)}{\Delta J^{s}_{\text{CDWS}}(eV = \pm 2\Delta)} = \left(\frac{\mu}{1-\mu}\right)^2 \frac{\Delta}{D}, \quad (26)$$

which, taking into account that $\mu < 1$ and $\Delta < D$, makes the feature at $eV = \pm 2D$ negligible.

Moreover, averaging over Σ_0^* almost completely obliterates the singularity at $eV = \pm 2D$. At the same time, it transforms the significant square-root singularities at $eV = \pm(D + \Delta)$ into humps. This result does not depend substantially on the subtleties of the averaging. As for the superconducting gap-edges at $eV = \pm 2\Delta$, the averaging procedure, which gave reasonable results for CDWS–I–N junctions, becomes ambiguous. In this case, the shape of the CVC depends on the order of operations, namely current differentiation and averaging over some spatially fluctuating parameter. To fit the experimental data for $G^{s}(V)$, obtained by the ac modulation method (see, e.g., [134]), one should act as in experiments, where the experimental tunnel current data are first averaged over a certain voltage interval and then are differentiated. Otherwise, the δ-like jump in $G^{s}(V)$ will be entirely lost. Therefore, current averaging has to be carried out before differentiation. Naturally, we have chosen this sequence of operations. It should be noted that CDW manifestations—hidden, but for dip–hump structures, below T_c by superconducting gapping—completely determine the $G(V)$ depletion above T_c, which is observed experimentally [1, 2, 5].

The appearance of the T-driven zero-bias peaks is a salient feature of certain CVCs displayed in figure 4. As is well known [116], this peak is caused by tunneling of thermally excited quasiparticles between empty states with an enhanced DOS located above and below equal superconducting gaps in symmetric S–I–S junctions. Such a feature was found, for example, in $G(V)$ measured for grain-boundary symmetric tunnel junctions in epitaxial films of the s-wave oxide CDWS $Ba_{1-x}K_xBiO_3$ [135]. One should be careful not to confuse this peak with the dc Josephson peak restricted to $V = 0$, which is often seen for symmetric high-T_c junctions [1]. The distinction consists in the growth of the quasiparticle zero-bias maximum with T up to a certain temperature, followed by its drastic reduction. On the other hand, the Josephson peak decreases monotonously as $T \to T_c$.

The more or less conventional zero-bias peaks in the CVCs of CDWS–I–CDWS junctions, involving electrodes with s-wave-like superconducting order parameters and studied here, have nothing to do with similar looking features of tunnel CVCs for high-T_c oxides revealed by STM in the non-symmetric set-up [124]. The latter most probably indicate the existence of surface bound states at the boundary between a normal probe and a d-wave superconductor. For the latter case, the peak disappears with growing T in the superconducting state, which has been shown, for example, by STM measurements of $La_{1.88}Sr_{0.12}CuO_4$ thin films [25].

The profile and the behavior of the zero-bias peak at non-zero T can be explained in our case by the fact that, in effect, owing to the non-homogeneity of CDWS electrodes, the junction is a combination of good numbers of *symmetric and non-symmetric* junctions with varying gap parameters. The former make up a mutual contribution to the current in the vicinity of the $V = 0$ point, and the width of this contribution along the V-axis is governed by temperature alone. On the other hand, every junction from the latter group gives rise to an elementary current peak in the CVC at a voltage equal to the relevant gap difference. All such elementary contributions form something like a hump around the zero-bias point, and the width of this hump along the V-axis is governed by the sum of actual—dependent on the zero-temperature values and on the temperature itself—gap spreads in both electrodes. It is clear that the T-behavior of the current contribution of either group is rather complicated, to say nothing of their combination.

From our CVCs calculated for both non-symmetric (figure 2) and symmetric (figure 4) junctions, it comes about that the 'dip' is simply a depression between the hump, which is mainly of CDW origin, and the superconducting coherent peak. Therefore, as has been noted in [136], the dip has no separate physical meaning. It disappears as the temperature increases because the coherent peak forming the other shoulder of the dip fades down, so that the former dip, by expanding to the $V = 0$ point, becomes an integral constituent of the shallow pseudogap minimum.

Averaging over the scatter of Σ_0^* is crucial for explaining the experimental data, whereas the analogous spread of Δ_0^* turns out to be much less significant. This point has been especially well recognized after the contributions of both gaps in the STM spectra of $(Bi_{0.62}Pb_{0.38})_2Sr_2CuO_{6+x}$ were separated by an ingenious trick [137]. Specifically, those authors normalized the measured local conductances by removing the inhomogeneous background of the larger gap. Then, it became clear that the superconducting order parameter is more or less homogeneous over the sample's surface, whereas the larger gap (corresponding to the dielectric order parameter) is essentially inhomogeneous, which is in full accordance with our results for BSCCO [138]. One should mention the alternative treatment [139] of cuprate inhomogeneities, namely, both large and small gaps, where they are considered as superconducting ones. On the contrary, we think that the larger energy scale DOS depletion beyond any doubt has a CDW origin because, for example, it appears both below and above T_c. At the same time, spatial inhomogeneity of the pseudogap (CDW) manifestations, treated here phenomenologically as a spread of Σ_0^*, might be connected to random dopant distributions, constituting a percolative network for example [39, 140, 141].

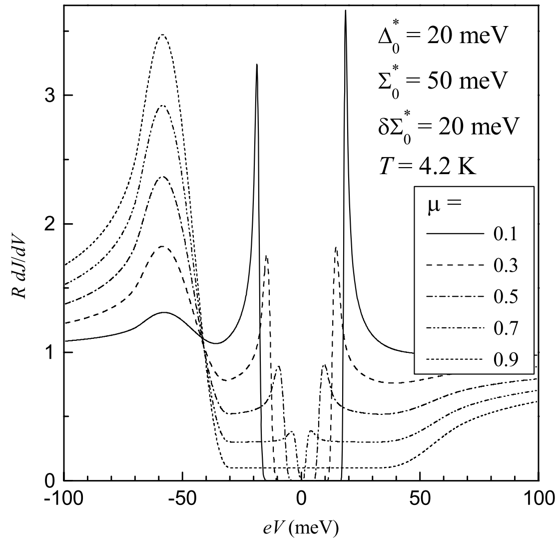

Figure 5. The same as in figure 2(a), but at a fixed temperature $T = 4.2$ K and for various values of the dielectric gapping parameter μ.

The value of the order of 0.1 for the control parameter μ was chosen to approximate the experimental data for BSCCO by theoretical CVCs. Raman investigations [142] for the oxide $La_{2-x}Sr_xCuO_4$, possessing the maximal $T_c \approx 42$ K for $x = 0.15$, also demonstrate that this parameter is equal to about 0.15. Nevertheless, it is instructive to analyze what would be the form of the CVCs if μ became substantially larger, i.e. if the dielectric gapping covered a larger FS region. The representative set of curves is depicted in figure 5. It is clearly seen how the hump transforms into a pronounced peak, while superconductivity is gradually suppressed by the shrinkage of the ungapped (n) FS sections.

We believe that such a scenario is appropriate to high-T_c cuprates; earlier, its validity was clearly demonstrated for superconducting dichalcogenides, for example [10]. In particular, the CDW semiconductor $TiSe_2$ has recently been shown to reveal superconductivity when n FS sections emerge with Cu doping (μ falls from 1 to lower values) [143]. These changes were directly detected by the appearance and steady growth with x of the Sommerfeld electronic specific heat coefficient γ_S for the solid solutions Cu_xTiSe_2. It is remarkable that both angle-resolved photoemission measurements and electron spectrum calculations for undoped 1T-$TiSe_2$ demonstrate [144] that just the excitonic-insulator scenario [37, 95] of the general CDW scheme is realized in this material.

Generally speaking, a close resemblance between the electronic properties of dichalcogenides and cuprates was noticed long ago (see, e.g., [73] and especially [30]). This seems natural due to their electronic two-dimensionality. Nevertheless, only modern ARPES measurements unequivocally demonstrated that the pseudogap in cuprates and that observed in dichalcogenide layered materials—for example 2H-$TaSe_2$ with its two successive CDW phase transitions, and, hence, undoubtedly being of the FS-nesting origin—are in close relation to each other [145]. It should be noted that the CDW mechanism in dichalcogenides is driven by the electron–phonon mechanism [145, 146], which, therefore, might be very significant for CDWs and superconductivity in cuprates. At the same time, the very phenomenon of CDW gapping (pseudogapping) in high-T_c oxides is rather complicated owing to a non-monotonic pseudogap versus temperature dependence [147], which requires that a generalization of the existing basic theoretical approaches be done.

7. The problem of negative conductance

It stems from figure 5 that for any μ the differential conductance $G(V) > 0$. This is a consequence of the aforementioned circumstance that the dip is merely a valley between the CDW hump and the coherent peak. This agrees with the overwhelming majority of data for high-T_c oxides, but for a few optimally doped BSCCO samples with T_c = 95 K [48, 148]. Both publications are from the same group and it seems that the data might refer to the same sample. In any case, notwithstanding the scarcity of the negative $G(V)$ observations, they should be carefully analyzed, since this result became a basis of the interpretation of the dip–hump structure [55] as a strong-coupling feature related to the spin resonance bosonic mode [56].

First of all, it should be indicated that negative differential tunnel conductance $G(V)$ is not forbidden at all by any general physical law. Moreover, just after the discovery of single-electron tunneling by Giaever [149], it was observed and understood that, in S–I–S junctions, the dependence $J(V)$ may have a descending portion [150, 151]. In the dip–hump structure region, a negative $G(V)$ may be obtained, in particular, in the framework of the strong-coupling-resonance scenario [56], where tunneling with anisotropic tunnel matrix elements $T_{\mathbf{pq}}$ is coherent [118–120]. Nevertheless, even the authors of [56] themselves, in order to fit the available data for tunneling across junctions involving BSCCO, are inclined to accept the idea of incoherent tunneling, where $G(V) > 0$. All such phenomena may be caused either by various modifications of the bare DOSs by many-body effects or by a momentum dependence of the tunnel matrix elements due to the electronic band-structure peculiarities or electron–electron correlations. Hence, the observation $G(V) < 0$, if any, might be considered as an intrinsic effect.

On the other hand, tunnel CVCs are well known to be prone to extrinsic thermal effects, which depend on the magnitude of the current density [152]. For instance, localized overheating was detected long ago by Giaever in a Pb–Ge–Pb–O–Pb layered system, where the germanium layer was penetrated by lead bridges [153]. A propagation of the normal zone across the sample of superconducting ceramics $BaPb_{1-x}Bi_xO_3$ warmed by the Joule effect was directly observed and shown to induce a thermal switching between the Josephson and the quasiparticle CVC branches [154, 155], which mimics the intrinsic multiple switching [154–159]. Therefore, a short-pulse technique was applied to distinguish between extrinsic and intrinsic effects [154, 155]. A similar technique was applied later to study interlayer superconducting tunneling on BSCCO mesas [57, 160–162]. The latter activity was connected to the danger of overheating in BSCCO mesa structures that contain stacks of several tunnel junctions.

The problem has been recently recognized [163–166] and has invoked hot debates on how to discriminate between superconducting gap and pseudogap manifestations [23, 167–170]. It should be noted that both typical pseudogap features [57, 162, 171–174] and dip–hump structures [162, 171, 172] were proved not to be artifacts and survived the application of the short-pulse method to mesas.

There is another method of self-heating compensation, if not complete elimination [175]. It includes a thermal feedback involving temperature monitoring and lowering the substrate-holder temperature as the mesa T grows due to the release of Joule heat. Application of this technique to BSCCO mesa natural tunnel junctions showed [13] that pseudogap phenomena survived the elimination of heating, thus being an intrinsic feature for at least this material. One would expect that heat flow outwards is easier in break junctions. Nevertheless, it is necessary to bear in mind the danger of overheating in break junctions as well.

Despite the existence of pseudogap features being confirmed, it was shown that their form and position substantially depend on the pulse duration [162], most likely due to the heating processes. Hence, the appearance of the negative $G(V)$ portion [48], which points to the fact that the tunnel current starts to drop with voltage growth, might originate from the unusual thermal increase of the resistance of the tunnel junction with V. This trend is the reverse of the R reduction due to a trivial thermo-field emission [176]. It also seems strange that, in spite of usually gradual changes of high-T_c oxide properties [5, 40, 177], such an effect was found by one group only and for one (optimal) oxygen-doping composition only. But should this effect be reproduced, the question would arise: what is principally peculiar for optimally doped BSCCO samples as compared with under- and overdoped ones, which provokes negative CVC portions? Unfortunately, there are not enough tunnel experimental data for BSCCO samples with doping close to the optimal one.

Of course, all the aforesaid does not disprove from the outset the usage of the strong-coupling idea [48, 49, 55, 56] to treat the dip–hump structure. However, this hypothesis has another shortcoming: it cannot explain the commonly encountered appearance of the dip–hump structures at only one voltage sign in CVCs for CDWS–I–N junctions involving various cuprates [45–47, 49, 64, 178–181]. The attempt [56] to invoke an additional *ad hoc* hypothesis that the Van Hove singularity plays a substantial role in the formation of CVC asymmetry does not seem persuasive, because then asymmetry becomes obligatory for a certain material, which is not always the case [50, 51].

While considering dip–hump structures—either well developed or less conspicuous—that were found in CVCs for various cuprates, one should bear in mind that, in the framework of the strong electron–boson interaction scenario, the bosons involved may turn out to be good old phonons [43, 60, 182–187]. In the case of BSCCO, this viewpoint is supported by STM measurements [43], which revealed that the conjectural boson feature does not depend on the oxygen stoichiometry δ, while the inhomogeneous gap distribution significantly shifts with δ. This is quite natural for crystal-lattice vibrations, whereas different magnetic excitation modes are strongly dependent on doping [188, 189]. Moreover, the boson peak energy obtained in [43] is equal to 52 meV, which is different from the spin-resonance-mode energy of 43 meV inferred from neutron scattering data [189].

8. Conclusions

To summarize, we have shown that the CDW manifestations against the non-homogeneous background can explain both subtle dip–hump structure structures in the tunnel spectra for high-T_c oxides and large pseudogap features observed both below and above T_c. The dip–hump structure is gradually transformed into the pseudogap-like DOS lowering as the temperature increases. Hence, both phenomena are closely interrelated, being in essence the manifestations of the same CDW-governed feature smeared by CDWS inhomogeneity. Therefore, one should not try to explain dip–hump structures and pseudogaps separately. The dependences of the calculated CVCs on the CDW phase φ fairly well describes the variety of asymmetry manifestations in the measured tunnel spectra for BSCCO and related compounds.

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