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Wavefront Motion in the Vicinity of a Phase Dislocation: "Optical Vortex"

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Abstract—In the scalar approximation, an analysis is made of the light field structure in the vicinity of a line of the ring phase dislocation corresponding to the zero value of the field formed by the interference of two uniaxial Gaussian beams. The formation of an "optical vortex" or the toroidal motion of a portion of a light flow around a ring phase dislocation is shown. © *2000 MAIK "Nauka/Interperiodica"*.

INTRODUCTION

A new term "optical vortex" (OV) has recently appeared in modern optics it combines the concept of a screw wavefront dislocation [1, 2] and an optical vortex proper as a field state in an active medium of a laser cavity that is characterized by the azimuthal dependence of phase [3]. A typical example of an OV is the Laguerre–Gaussian mode with a nonzero azimuthal index [4]. The necessary condition of OV existence is that the wave field amplitude vanishes on a certain line representing the vortex axis [1, 5]. The zero-amplitude line may coincide with the axis of a light beam or have a more complex form [6]. Of particular interest is the case where the OV axis is perpendicular to the direction of beam propagation. In this case, the "edge" wavefront dislocation is formed. The formation of an edge dislocation due to the interference of two Gaussian beams will be considered below.

RING PHASE DISLOCATION

It is known that zero-amplitude lines of the summary light field can be formed by the interference of optical beams. The coherent superposition of two monochromatic linearly polarized waves gives zero amplitude in the case where the interfering beams have the same amplitudes and their phase difference is equal to π, which provides destructive interference. The formation of zero-amplitude rings for two uniaxial Gaussian beams was analyzed in [7], where the conditions of their formation were found. The aim of this paper is to analyze the structure of the optical wavefront in the vicinity of a zero-amplitude line of the summary field and study an OV in this region.

To simplify the statement of the problem, let us consider in cylindrical coordinates ρ, z two monochromatic (with frequency ω) Gaussian beams ($E_1(\rho, z)e^{-i\omega t}$ and $E_2(\rho, z)e^{-i\omega t}$) whose axes coincide with the z-axis and whose waists are positioned at $z = 0$. In this plane, both beams have a plane wavefront, and the summary field amplitude has the form

$$E(\rho) = E_1 \exp\left(-\frac{\rho^2}{w_1^2}\right)\exp i\Phi_1 + E_2 \exp\left(-\frac{\rho^2}{w_2^2}\right)\exp i\Phi_2, \quad (1)$$

where E_1 and E_2 are the amplitude parameters of the beams ($E_1 > E_2$), w_1 and w_2 are their transverse dimensions at an amplitude level of $1/e$ ($w_1 < w_2$), and Φ_1 and Φ_2 are their phases. To localize the zero-amplitude ring in the plane $z = 0$, let us impose the condition $\Phi_1 - \Phi_2 = \pi$; i.e., the interference of the beams is destructive. In this case, the zero-amplitude ring is determined by the expression

$$\rho_0 = \sqrt{\frac{w_1^2 w_2^2}{w_2^2 - w_1^2}\ln(E_1/E_2)}. \quad (2)$$

Prior to the deduction of an expression for the phase of the summary field at $z \neq 0$, we note that the phases of the plane wavefront inside the ring of radius ρ_0 and outside it differ by π; i.e., we obtain a phase jump by π on the zero-amplitude line, which leads to a phase defect of the wavefront or the edge phase dislocation. On exit from the plane $z = 0$, the condition of stationary destructive interference is violated and the mutual cancellation of beams is no longer observed. The condition of formation of next ring dislocations can be obtained according to the analysis made in [7], taking into account the Gouy phase for Gaussian beams.

WAVEFRONT FORM IN THE VICINITY OF A RING DISLOCATION

Because the propagation of each beam is described by the known expression [4], the phase Φ of the sum-

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mary field is determined as

$$\tan\Phi = \frac{\mathrm{Im}[E(\rho, z)]}{\mathrm{Re}[E(\rho, z)]} = \frac{E_1(\rho, z)\sin\left(\Phi_1 + kz + \frac{k\rho^2}{2R_1(z)} - \arctan\frac{z}{L_1}\right) + E_2(\rho, z)\sin\left(\Phi_2 + kz + \frac{k\rho^2}{2R_2(z)} - \arctan\frac{z}{L_2}\right)}{E_1(\rho, z)\cos\left(\Phi_1 + kz + \frac{k\rho^2}{2R_1(z)} - \arctan\frac{z}{L_1}\right) + E_2(\rho, z)\cos\left(\Phi_2 + kz + \frac{k\rho^2}{2R_2(z)} - \arctan\frac{z}{L_2}\right)}, \tag{3}$$

where $E_{1,2}(\rho, z) = \frac{E_{1,2}}{\sqrt{1 + z^2/L_{1,2}^2}}\exp\left[-\frac{\rho^2}{w_{1,2}^2(1 + z^2/L_{1,2}^2)}\right]$ are the beam amplitudes, $R_{1,2}(z) = z(1 + L_{1,2}^2/z^2)$ are the radii of curvature of the wavefronts, $L_{1,2} = kw_{1,2}^2/2$ are Rayleigh lengths, Gouy phases are equal to $-\arctan(z/L_{1,2})$, and k is the wave number.

By taking into account the fact that the beams are opposite in phase at $z = 0$ ($\Phi_1 - \Phi_2 = \pi$), one can rewrite expression (3) in the form

$$E_1(\rho, z)\sin\left(\Phi_1 - \Phi + kz + \frac{k\rho^2}{2R_1(z)} - \arctan\frac{z}{L_1}\right) = E_2(\rho, z)\sin\left(\Phi_1 - \Phi + kz + \frac{k\rho^2}{2R_2(z)} - \arctan\frac{z}{L_2}\right). \tag{4}$$

Thus, equation (4) implicitly specifies the form of the wavefront Φ = const, which was used to construct the set of wavefronts showing the motion of the wave near the waist plane. Figure 1 presents the distributions of beam amplitudes in the waist and the summary amplitude (at the point ρ_0, the amplitude vanishes and the phase changes by π). Moreover, it presents on a rough scale the shape of isophase lines on both sides of the waist. The lines of wave nodes, i.e., $\Phi = \pm\pi/2, \pm 3\pi/2$, where taken as isophase lines. One can see that the passage through the wave dislocation leads to the loss of one wave period inside the dislocation ring and inverts a protrusion on the wavefront. The choice of beam parameters $w_1 = 10$ and $w_2 = 100$ (transverse dimensions in the waist normalized by $k = 1$) and the corresponding Rayleigh lengths $L_1 = 50$ and $L_2 = 5000$ satisfies the beam paraxiality condition $L_{1,2} \gg 1/k$.

By varying the parameter Φ_1, one can follow the passage of the wavefront through the waist plane. The same pattern of wavefront cross sections can be obtained as set of isophase lines Φ = const for $\Phi_1 = 0$. The form of isophase lines near the dislocation is shown in greater detail in Fig. 2a. The tracing along a closed circuit around the point corresponding to the section of an edge dislocation causes a change of phase by 2π. A part of the line terminates at this point, which is evidence of the wavefront discontinuity. For $\Phi = 0$, one observes the branching of an isophase line at the point specified by a circle.

As the wave passing through a ring dislocation travels further, the wavefront in the region $z > 0$ propagates not only along the z-axis, but in the transverse direction as well. Figure 2b shows the wavefront normals along which the wavefront locally moves. In a certain region, the wavefront normal has a component that is directed toward the motion of the wave as a whole. In other words, a part of the wavefront rotates about the dislocation line and forms an optical vortex, which is well known in the case of a screw dislocation of the light field with the axis parallel to the direction of wave propagation [1, 5, 6]. In contrast to the "longitudinal" optical vortex, the domain of phase rotation about the zero-amplitude line for the "transverse" vortex represents a small bounded region of space, which is considerably smaller in size (for real beams) than the wavelength $\lambda = 2\pi/k$.

Figure 3 illustrates the process of motion in the counter direction for the wavefront passing through the plane $z = 0$. The wavefront bifurcation point $\rho = \rho_s$ is specified by the empty circle, and the section of the ring dislocation $\rho = \rho_0$ is specified by the bold circle. As a succession of wavefronts passes through a ring dislocation, the top of a smooth wavefront is "transferred" to the neighboring one having a cut in the form of a circle of radius ρ_0. The process includes the combination of two surfaces, subsequent separation, and exchange of parts. The wavefront in the upper half plane accepts the circle of radius ρ_s, encloses the cut, and gives to the lower one the ring found between ρ_s and ρ_0.

The equation for the determination of the bifurcation point corresponding to the branching of the isophase point $\Phi = 0$ or to the phase saddle point ρ_s can be obtained from equation (4) in the form

$$k + \frac{k\rho^2}{2L_1^2} - \frac{1}{L_1} = \frac{E_2(\rho)}{E_1(\rho)}\left(k + \frac{k\rho^2}{2L_2^2} - \frac{1}{L_2}\right). \tag{5}$$

The version described above for the ring dislocation formation by the interference of two uniaxial Gaussian beams models real cases of zero-amplitude line formation, namely, the beam focusing with an aspherical (Gaussian) lens [8] and the formation of Airy rings in the focus of a lens bounding the transverse beam size [9, 10]. As will be shown below, the motion of wave-

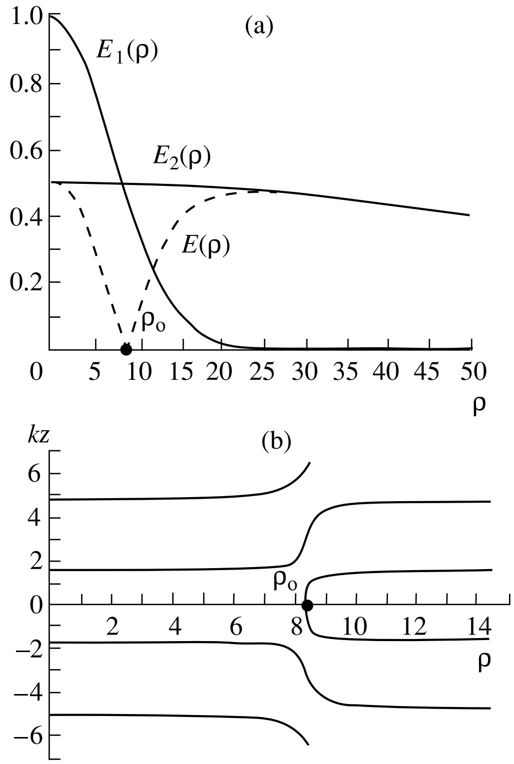

Fig. 1. (a) Amplitude distributions for the interfering Gaussian beams $E_1(\rho)$ and $E_2(\rho)$ and the summary field $E(\rho)$ in the beam waist ($z = 0$) for the beam parameters $E_1 = 1$, $E_2 = 0.5$, $w_1 = 10$, and $w_2 = 100$ (in units normalized by $k = 1$). The amplitude $E(\rho)$ vanishes on the ring of radius ρ_0. (b) The form of isophase lines $\mathrm{Re}[E(\rho, z)] = 0$, $\Phi_1 = 0.1$.

fronts may qualitatively change depending on the relation between beam amplitudes and, therefore, depending on the radius of a ring dislocation. These two cases are topologically different and one may assign to a ring dislocation a topological charge of sign plus or minus.

The topological charge may be also introduced in a different way. It is based on the phase gradient around the zero-amplitude point in the longitudinal section of a beam [11]. However, in this case, the concept of a topological charge of a ring dislocation as a whole object is lost.

PROPAGATION OF POYNTING'S VECTOR IN THE VICINITY OF A PHASE DISLOCATION

The wavefront motion, which was considered above, is a result of circulation of the light energy flow about a dislocation line. Within the framework of the paraxial approximation used here, the calculation of Poynting's vector $\mathbf{S} = \mathrm{Re}[(\mathbf{E} \times \mathbf{H}^*)/2]$, where $\mathbf{E} = \hat{\mathbf{y}} E(x, y, z)$ and $\mathbf{H} = \frac{1}{i\omega\mu_0}\mathrm{rot}\mathbf{E}$, for the wave linearly polarized along the y-axis leads to the following expressions for the radial and axial components in the plane x, z:

$$S_x = \frac{1}{2\mu_0\omega}\Bigg\{|E_1(x,y,z)|^2\frac{kx}{R_1(z)} + |E_2(x,y,z)|^2\frac{kx}{R_2(z)} + |E_1(x,y,z)E_2(x,y,z)| \times\left[kx\frac{R_1(z)+R_2(z)}{R_1(z)R_2(z)}\cos\Delta\Phi + 2x\frac{w_2^2 - w_1^2}{w_1^2 w_2^2}\sin\Delta\Phi\right]\Bigg\}, \tag{6}$$

$$\begin{aligned} S_z = \frac{1}{2\mu_0\omega}\Bigg\{&|E_1(x,y,z)|[|E_1(x,y,z)| + |E_2(x,y,z)|\cos\Delta\Phi]\left(k + kx^2\frac{L_1^2 - z^2}{2(L_1^2+z^2)^2} - \frac{L_1}{L_1^2+z^2}\right) \\ &+ |E_2(x,y,z)|[|E_2(x,y,z)| + |E_1(x,y,z)|\cos\Delta\Phi] \times\left(k + kx^2\frac{L_2^2 - z^2}{2(L_2^2+z^2)^2} - \frac{L_2}{L_2^2+z^2}\right) \\ &+ |E_1(x,y,z)E_2(x,y,z)|\left(-\frac{z}{z^2+L_1^2} + \frac{z}{z^2+L_2^2} + \frac{2x^2L_1^2 z}{w_1^2(z^2+L_1^2)^2} - \frac{2x^2L_2^2 z}{w_2^2(z^2+L_2^2)^2}\right)\sin\Delta\Phi\Bigg\}, \end{aligned} \tag{7}$$

where the phase difference of the interfering beams is given by

$$\Delta\Phi = \frac{kx^2}{2}\frac{R_2(z) - R_1(z)}{R_1(z)R_2(z)} - \arctan\frac{z}{L_1} + \arctan\frac{z}{L_2} + \pi. \tag{8}$$

In the plane $z = 0$, the radial component vanishes because $R_{1,2}(0) = \infty$ and $\Delta\Phi = \pi$. The axial component vanishes at $x = x_0$, i.e., on the dislocation line where the field amplitude vanishes and, moreover, under the condition

$$k + \frac{kx^2}{2L_1^2} - \frac{1}{L_1} = \frac{E_2(x)}{E_1(x)}\left[k + \frac{kx^2}{2L_2^2} - \frac{1}{L_2}\right], \tag{9}$$

which coincides with the equation obtained above for a phase bifurcation point. On the interval between two zero values, the axial component of Poynting's vector has a negative value, and positive values are obtained outside the interval. The fact that Poynting's vector vanishes at the phase bifurcation point corresponds to the energy flow stagnation, i.e., the formation of a standing wave between the normal and circular flows. The distance between the stagnation point and the zero-

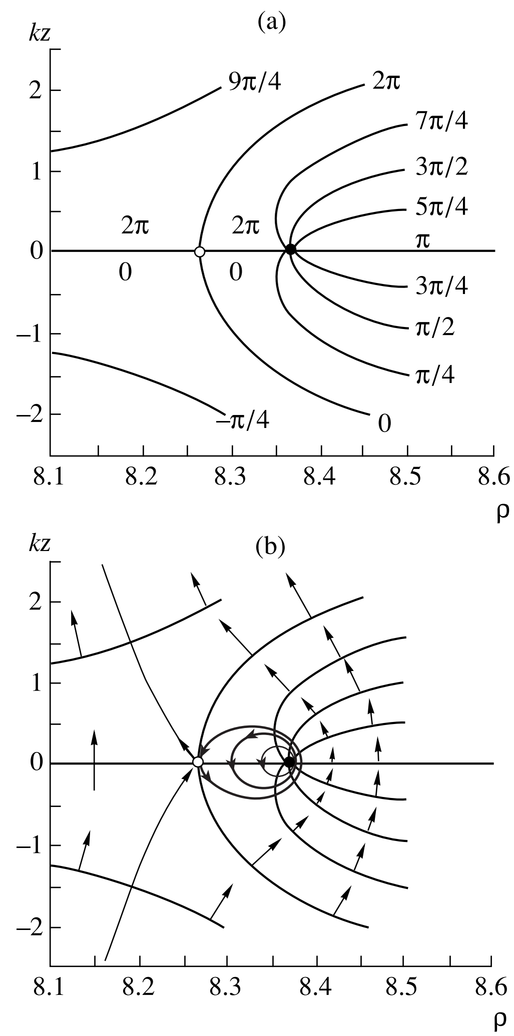

Fig. 2. (a) Set of isophase lines with a step of $\pi/4$ in the vicinity of an edge dislocation. The point of zero field amplitude is specified by the bold circle, and the isophase lines terminate at this point. The branching point of the isophase line $\Phi = 0$ is specified by the empty circle. (b) The structure of wave normals (specified by arrows) in the vicinity of an edge dislocation. The continuous line represents the separatrix separating the flows inside the ring dislocation and outside it. The separatrix forms a closed ring around the zero-amplitude point.

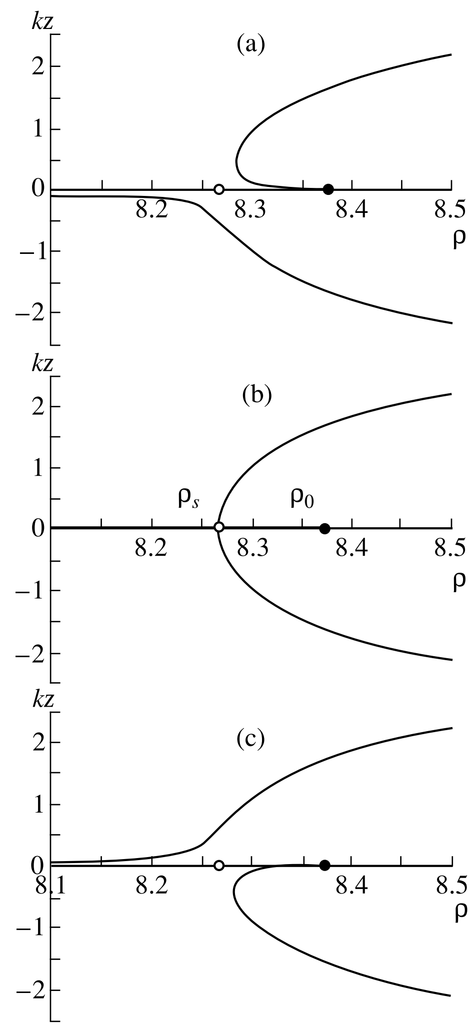

Fig. 3. (a) Wavefront in the half plane $z < 0$, when approaching the beam waist, has a smooth surface with a protrusion in the direction of wave propagation; the wavefront in the half plane $z > 0$ has a discontinuity on the ring $\rho = \rho_0$. (b) Form of the wavefront in the waist. In the saddle point, which is specified by the empty circle, branching takes place. (c) The wavefront passed through the waist becomes smooth. In this case, a discontinuity of the wavefront in the region positioned upstream from the waist is formed. As the wavefront moves along the z-axis, a part of its surface displaces from the upper half plane to the lower one.

field point can be obtained from transcendental equation (9). For $L_2 \gg L_1$, one can obtain the estimate

$$x_s - x_0 \approx (x_0^2 - w_1^2)/k^2 w_1^2 x_0. \tag{10}$$

From (10), it follows that the distance between these two points increases with increasing distance from the dislocation ring to the beam waist and that the stagnation point may be found both inside the dislocation ring and outside it.

Figure 4 shows the field of Poynting's vector directions in the vicinity of the edge dislocation, which was calculated on the basis of formulas (6) and (7). The pattern obtained for the energy flow qualitatively agrees with the pattern shown in Fig. 2b. The flow bends around the dislocation region and forms a circular motion in the direct vicinity of the dislocation. A typical size of the region occupied by the circular flow is 10^{-2} (in wavelengths) along the transverse coordinate and 10^{-1} along the longitudinal coordinate.

NUMERICAL SIMULATION OF THE EFFECT

To verify the results obtained in the paraxial approximation, we directly numerically solved the wave equation with the boundary conditions corresponding to the

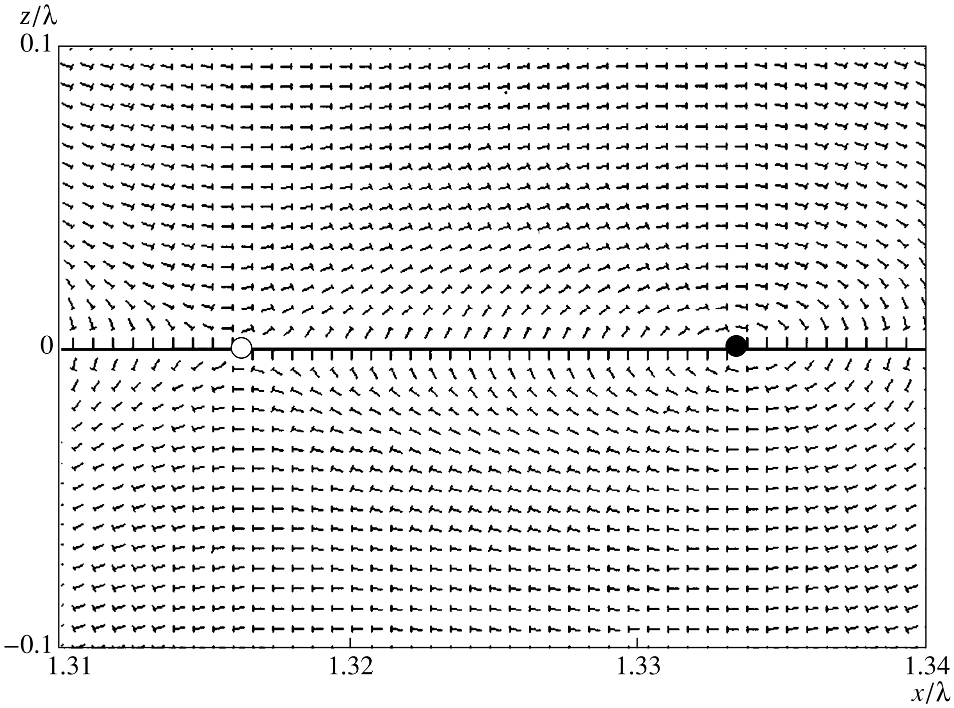

Fig. 4. The field of Poynting's vector directions calculated in the vicinity of the edge dislocation whose section is specified by the bold point. The position of the stagnation point is specified by the empty circle. The origin of each vector has a perpendicular mark.

known distribution of amplitudes of the beams in the plane $z = 0$ and their phase velocities. The axial symmetry makes it possible to simplify the calculation by making a change to the cylindrical coordinate system. This gives

$$\rho\frac{\partial^2 E}{\partial z^2} + \rho\frac{\partial^2 E}{\partial \rho^2} + \frac{\partial E}{\partial \rho} = -\rho k^2 E.$$

This equation was numerically solved for a depth of one wavelength, which made it possible to follow in detail the variation of the wavefront form. As a line of equal phase, we chose the boundary between positive and negative values, i.e., the section of the surface of wave nodes. The results obtained verified the analytical calculation of the motion of isophase lines in the course of wave passage through a ring phase dislocation. However, one should take into account that the boundary conditions used by us may be different from the real conditions, which will be discussed below.

DISCUSSION OF THE RESULTS AND CONCLUSIONS

The condition of the paraxial approximation in which the expressions used for Gaussian beams are valid imposes restriction on the accuracy of the results obtained because the events under consideration take place on distances smaller than one wavelength. Because of this, let us analyze a possible effect of the error caused by the approximate description of the beams.

The use of the scalar wave equation eliminates the effect of the longitudinal component of the electric field, which can change the character of the events considered here. However, the effect is retained for a "two-dimensional" Gaussian beam $E(x, z)$ in which the longitudinal field component is absent, and in the case under consideration, a ring dislocation is transformed into two linear dislocations with a similar behavior of the energy flow around them.

The difference of the paraxial approximation from the exact (vector) solution may lead to a non-Gaussian profile of the amplitude distribution in the beam waist. However, as shown in [11], noticeable distinctions are observed for a very sharp focusing ($w \approx 1/k$), which is inaccessible in a real experiment. Because of this, for the case under consideration where both beams satisfy the requirement of paraxiality, the difference of the profile of the amplitude distribution from the Gaussian profile may be thought of as insignificant. The second difference of the exact solution from the paraxial one consists in the description of the wavefront form. The paraxial approximation gives a veritably incorrect result for $\rho \gg w_0$ because the wavefront asymptotically approaches the waist plane when $\rho \longrightarrow \infty$. Nevertheless, in the region under consideration $\rho \le w_0$, the paraxial approximation gives the result that is correct both qualitatively and quantitatively. The phase velocity on the beam axis increases with respect to value for the plane wave (Gouy effect [12]). It is precisely the difference of phase velocities of the interfering waves that is a physical reason of OV formation. The phase velocities of the beams in the direct vicinity of the waist can be expressed in the form

$$\nu_{1,2} = c\left(1 + \frac{\rho^2}{2L_{1,2}^2} - \frac{1}{kL_{1,2}}\right)^{-1}, \qquad (11)$$

where $c = \omega/k$ is the speed of light. On the axis of each beam, the phase velocity exceeds c, and it becomes equal to c on the ring whose radius is equal to the size of the beam waist. In the case of interference of uniaxial beams, their phase velocities in the waist plane have equal values on the ring of radius ρ_ν:

$$\rho_\nu = \left[\frac{2L_1L_2}{k(L_1 + L_2)}\right]^{1/2} = \left[\frac{w_1^2 w_2^2}{w_1^2 + w_2^2}\right]^{1/2}. \qquad (12)$$

The OV structure is determined by the relation between ρ_0 and ρ_ν. For $\rho_0 < \rho_\nu$, the phase velocity in the vicinity of a ring dislocation is higher inside this region than outside it. As a result, the flow is twisted "toward the interior" of the ring dislocation in the way shown in Fig. 4 (the radius of the saddle point is smaller than the dislocation radius). For $\rho_0 = \rho_\nu$, the velocities become equal, and the saddle point coincides with the dislocation point. At this moment, two new ring dislocations are formed, which are positioned on both sides of the plane $z = 0$. Finally, for $\rho_0 > \rho_\nu$, the saddle point is positioned outside the dislocation ring and the flow circulating about the dislocation line reverses its direction.

One can observe the effect in the experiment with the aid of the method of a probing particle [13] by using submillimeter waves. A particle of micrometer size, when positioned in the region of an OV, will rotate about the vortex axis (for example, in a cavity with a liquid). One can detect this rotation with the aid of a microscope by using illumination with weak incoherent radiation.

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