Charge-density-wave origin of the dip-hump structure in tunnel spectra of the BSCCO superconductor

Alexander M. Gabovich* and Alexander I. Voitenko†
Institute of Physics, National Academy of Sciences of Ukraine, Nauka Avenue 46, Kyiv 03680, Ukraine
(Received 4 December 2006; revised manuscript received 15 January 2007; published 28 February 2007)

Differential conductance G as the function of the bias voltage V across the tunnel junction between a normal metal and an inhomogeneous superconductor with charge density waves (CDW's) has been calculated by spatial averaging over random domains with varying superconducting- and normal-state properties. For these materials, irregularly distorted CDW patterns with spatially scattered values of various parameters were earlier shown to manifest themselves in a great body of experimental data. The results of the calculations were applied to explain the well-known dip-hump structure in the $G(V)$ dependence for $Bi_2Sr_2CaCu_2O_{8+\delta}$ and other high-T_c cuprates.

DOI: 10.1103/PhysRevB.75.064516 PACS number(s): 73.43.Jn, 71.45.Lr, 74.50.+r, 74.81.−g

I. INTRODUCTION

Tunnel and point contact spectroscopies are powerful tools for studying electron spectra in solids. In particular, while considering the quasiparticle differential conductance versus bias voltage dependences $G(V)$—hereafter coined as current-voltage characteristics (CVC's)—for tunnel junctions that involve Bardeen-Cooper-Schrieffer (BCS) superconductors, their peculiarities reflect the existence of superconducting energy gaps Δ in the BCS electrodes. The situation for junctions with anisotropic, e.g., d-wave superconductors is to a certain extent more complicated, since gap-driven features in the resulting densities of states (DOS) are smeared by averaging over the angle dependences of the superconducting order parameter (OP).[1]

The discrepancies arise when nonsymmetric SIN junctions (S stands for a superconductor, I for an insulator, and N for a normal metal) include high-T_c oxides. In such a case, $G(V)$'s lose their symmetricity with respect to the voltage V sign, so that the conventional equation

$$G(-V) = G(V) \tag{1}$$

becomes invalid. The most remarkable features of such a nonsymmetric behavior for the SIN junctions are different amplitudes of the coherent peaks in the positive- and negative-voltage branches and the appearance of the so-called dip-hump structures (DHS's)—one per each branch—that are located at biases larger than the coherent peak positions. The loss of the symmetry is observed in CVC's for junctions that include, e.g., $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BSCCO),[2–5] $Bi_{2-x}Pb_xSr_2CaCu_20_{8+\delta}$,[6] $Bi_2Sr_2Ca_2Cu_3O_{10+\delta}$,[7] $(Cu,C)Ba_2Ca_3Cu_4O_{12+\delta}$,[8] $Tl_2Ba_2CuO_{6+\delta}$,[5,9] $TlBa_2Ca_2Cu_2O_{10-\delta}$,[10] and $HgBa_2Ca_{n-1}Cu_nO_{2n+2+\delta}$ (Ref. 11) as a superconducting electrode. The body of experimental data is most abundant for junctions with BSCCO electrodes owing to high reproducibility of the results in this case. Therefore, below we shall mainly address just data obtained for the BSCCO oxide, although the DHS should be recognized as common for all hole-doped oxides.[10]

The majority of SIN junctions reveal a DHS only in one of the CVC branches. Nevertheless, there are also observed two symmetrically located DHS's (one per branch) but with amplitudes that can differ drastically.[10,12] The coherent peak and the DHS in the same branch are close enough, so that the difference in the DHS amplitudes or even the DHS emergence in only one branch might be coupled with the difference between the coherent peak amplitudes. At the same time, in the $G(V)$ dependences for symmetrical SIS junctions, these structures may appear in both voltage branches at the same $|V|$ biases and, if so, possess identical widths and amplitudes.

Unfortunately, we are not aware of any information in the literature concerning the $G(V)$ dependences for the nonsymmeric SIS junctions between *different* high-T_c oxides, although, as will become clear from what follows, such experimental data could be very useful in elucidating the origin of the $G(V)$ nonsymmetric behavior.

The conventional theory of tunneling in junctions involving superconductors brings about symmetrical CVC's, despite whether the junction is symmetrical or not or whether the superconductor is of the s or d type. A lot of theories have been proposed to explain such nonsymmetricity, but the problem has remained challenging since the moment of the first observations. The attempts to tackle it can be classified into four groups.

In the first group, the CVC nonsymmetricity is attributed to some unavoidable technological errors while carrying out the experiment. This possibility can be ruled out from the very beginning, because otherwise the reproducible results could be scarcely obtained in various experimental groups.

In the second group, there are theories that consider the electron subsystem of high-T_c oxides to be described by a non-Fermi-liquid theory. For instance, the resonating valence bond (RVB) theory[13] could explain the asymmetry.[14] But the very applicability of the RVB concept to cuprates has not been proven yet, so that this explanation seems to be rather exotic. Moreover, there are some specific difficulties originating from the absence of the electron-hole symmetry in the RVB-based models. Such kind of symmetry loss had to hamper Andreev reflections in junctions involving high-T_c cuprates, which is not the case.[15]

The rest of the theories couple the observable CVC nonsymmetricity with the inherent properties of the system, for which the Fermi liquid theory is considered valid. These theories are in turn divided in half: those which consider the violation of rule (1) *below* the critical temperature T_c of the superconducting electrode as a result of the quasiparticle properties just in the *superconducting* phase (group 3 of our classification) and those which regard the parameters of this electrode in the normal phase—i.e., *above* T_c—to be responsible for the phenomenon concerned (called group 4; see, e.g., Refs. 16–18). Any of the group-4 theories results in CVC nonsymmetricity below T_c because in this case "symmetrical" superconductivity emerges against the "nonsymmetrical" background. There is a lot of evidence (at least in BSCCO and related compositions) that the pseudogap feature, which we attribute just to the CDW, coexists with a superconducting gap below T_c within a wide doping range.[16,17,19–30]

Therefore, the latter viewpoint seems to us more plausible. Those who proclaim the former (group-3) one advance various indirect arguments in favor of their position.[31,32] In particular, the authors of Ref. 32 assumed that an interplay exists between the superconducting gap and the so-called pseudogap Π of the nonsuperconducting nature, both gaps having d symmetry. Then, using a specific procedure they demonstrated for the case of the symmetric junction that dips are not deep enough to make $G(V)$ negative at any V. At the same time, in some experiments such values were observed indeed.[2,33] Therefore, a conclusion was drawn[32] that the nonsuperconducting origin of the DHS can be excluded altogether. We stress that this result was obtained only in the approach of Ref. 32. Below we shall show that just the pseudogap of nonsuperconducting origin can be the cause of the nonsymmetric CVC's.

Another argument of the same experimental group is that, while doping a high-T_c superconductor, the positions of the superconducting gap Δ feature and the DHS correlate.[10,12,19,34] In this connection, it should be emphasized that doping can also influence the properties of the normal phase. From this point of view, it would be very instructive, in principle, to examine CVC's of tunnel junctions involving high-T_c oxides at temperatures T above T_c. They could be a real criterion of the validity of those theories, because in this case CVC's must be perfectly symmetrical above T_c. Unfortunately, information is rather scarce in this case and vague because all pseudogap-related features become smeared with T. Nevertheless, attention should be drawn to the fact that even for *symmetric* junctions some experimental CVC's involving BSCCO[35–37] and measured above the corresponding T_c are actually nonsymmetric. It comes as no surprise that nonsymmetric junctions demonstrate a similar asymmetry.[3]

Theories belonging to the fourth group, in their turn, are based on different models. For instance, a possible CVC asymmetry in BSCCO may be due to a van Hove singularity in the quasiparticle spectrum located not exactly at the Fermi level.[38] Additionally, the availability of the electron spectrum van Hove singularity might explain an apparent gap anisotropy of BSCCO, no matter what the symmetry of the superconducting OP is: d-wave or extended s-wave one.[39,40] The problem consists in the absence of an unequivocal proof that the van Hove scenario[11,40–43] is valid. Of course, other ways of electron-hole symmetry violation as an origin of the CVC asymmetry are also possible.[44]

Nevertheless, both the van Hove scenario and any fourth-group-related theory were refuted in Ref. 12 (see also a more detailed theory in Ref. 32), where it was shown experimentally that DHS's are possible in both CVC branches for SIN junctions. Instead, the authors of Refs. 12, 32, and 45–49 explain the presence of the DHS as a manifestation of strong-coupling *superconducting* effects, conspicuous due to the quasiparticle interaction with a certain boson mode. In particular, the emphasis has been made[45] on the likely spin-wave origin of this mode, which is, at the same time, the mediator in the d-wave Cooper pairing. But in this case the DHS must always be observed symmetrically in both CVC branches, with the whole CVC's for SIN cuprate-based junctions being symmetrical, which contradicts the experiment.[2–4,6–9,11,12] It should be noted that the coupling in cuprates is claimed[12,32,45–48] to be so strong that the signatures of the electron-boson interaction at energies $eV=\omega_0$, where $e>0$ is the elementary charge and ω_0 is the relevant mode frequency, are conspicuously observable not only in the second derivative d^2J/dV^2 of the tunnel current J, which is typical of conventional superconductors with an electron-phonon mechanism of superconductivity,[50] but even in the first derivative $dJ/dV \equiv G(V)$—as a deep DHS. The representation of a localized boson as a "defect," which strongly scatters electrons,[52,51] constitutes a theoretical basis for such views.

We consider that the DHS occurs due to the overlap of the coherent peaks induced by Cooper and pseudogap pairing. In this model, the hump is no more than the modified (smeared) peak caused by the pseudogap, while the dip is simply a transitional region (the "valley") between those two peaks. Below we demonstrate that this model can properly describe the experimental results. The first key point of our theory is the interpretation of the pseudogap as a gap in the quasiparticle spectrum, driven by charge density waves (CDW's). Such an interpretation is directly supported by observations of periodic structures in BSCCO taking advantage of various experimental methods.[53–66] Photoemission studies reveal the $4a_0 \times 4a_0$ charge-ordered "checkerboard" state (a_0 is a lattice constant) in $Ca_{2-x}Na_xCuO_2Cl_2$,[67] and tunneling measurements visualized the same kind of ordering in BSCCO.[66] A dynamical charge inhomogeneity probably connected to the stripe order was recently observed in $La_{2-x}Sr_xCuO_4$ (with $x=0.07, 0.15$).[68] There is also a lot of indirect evidence in favor of the important role of CDWs in cuprates.[16,17] From the theoretical point of view, one should mention a number of other studies in which CDW scenarios for different superconductors were suggested and developed.[23,29,59,69–74]

It is important to emphasize that both the bidirectional (checkerboard) and the unidirectional (stripelike) CDW's have much in common in the sense of how they influence superconductivity, although the microscopic origin of various kinds of charge ordering (phase separation) might be different.[29,40,59,75]

As for the competitive character of the relations between Cooper pairing and CDW's, it has been extensively studied theoretically and unambiguously proven experimentally, e.g., for such varying materials as $(Lu_{1-x}Sc_x)_5Ir_4Si_{10}$ (Ref. 76) and Cu_xTiSe_2.[77]

The self-consistent theory of CDW superconductors (CDWS's) was developed by us earlier[78] on the basis of the

Bilbro-McMillan model[79] for the superconducting transition in a partially gapped metal, with both gaps, superconducting Δ and dielectric Σ, having s symmetry. Although there is a predominant opinion that the superconducting gap in high-T_c oxides has d symmetry,[80–85] nevertheless, there are also experimental results and sound considerations (see, e.g., Refs. 86–90) that this issue still remains open. In particular, the authors of Ref. 91, inspired by an apparent success of the two-gap picture of superconductivity in MgB_2,[92] proposed a similar phenomenological two-gap $s+d$ scheme to explain the existence of the inflection point in the T dependence of the in-plane magnetic penetration depth for $La_{1.83}Sr_{0.17}CuO_4$.

Concerning the pseudogap, a discussion of its nature existed at the very beginning.[93] In any case, attempts to simulate even the principal features of the experimental $G(V)$ dependences assuming OP d symmetry face difficulties; i.e., it is impossible to describe simultaneously the $G(V)$ shape in the intragap region, the heights of coherent peaks at the gap edges, and the DHS features,[94] even if an additional angle weighting factor $g(\theta)$ is inserted into the expression for the tunnel DOS in an *ad hoc* manner.[7,32,94,95] Therefore, we have supposed isotropic s symmetry for both the superconducting (SOP) and dielectric (DOP) OP's and, according to our numerical results given below, the assumption turned out to be quite satisfactory from a practical point of view.

In this connection, we emphasize that the aim of the work is not to explain in detail the behavior of $G(V)$ in the whole bias voltage interval but rather to simulate both gaplike features. One of them is the DHS structure smeared by the intrinsic inhomogeneities of oxides (it is the second key point of the theory proposed). It should be mentioned that a substantial quenched disorder should affect the unidirectionality of different CDW-like structures,[96] a problem not touched by our semiphenomenological approach.

In this work, only nonsymmetric SIN junctions are studied. The justification of our approach in the case of SIS ones remains the same, but calculations turn out to be more involved. Hence, SIS junctions will be considered in a separate paper.

II. THEORY

A. CDW superconductors

The detailed formulation of the self-consistent theory for a homogeneous partially gapped CDWS can be found elsewhere.[78] Here, we shall point out its fundamental features relevant to the subject concerned.

The Fermi surface (FS) of the ungapped homogeneous CDWS [i.e., above both superconducting-transition, T_c, and CDW (structural-transition), T_d, critical temperatures] includes (i) two congruent ("nested," $i=1,2$) sections, where the degenerate (d) quasiparticle spectrum branches $\xi_{1,2}(p)$ reckoned from the common Fermi level are linked according to the relationship

$$\xi_1(\mathbf{p}) = -\xi_2(\mathbf{p}+\mathbf{Q}), \tag{2}$$

$\mathbf{Q}$ being the CDW vector, and (ii) the remaining part ($i=3$), where the quasiparticle spectrum $\xi_3(\mathbf{p})$ is nondegenerate (n). The extent of such an FS partition is described by a parameter

$$\mu = \frac{N_d(0)}{N_d(0)+N_n(0)}, \tag{3}$$

where $N_d(0)$ and $N_n(0)$ are the quasiparticle DOS at the d and n FS sections, respectively. The parameter μ does not change with temperature and by definition falls within the interval $0<\mu<1$ (partial gapping[79]). The CDWS Hamiltonian includes the interaction terms responsible for dielectric and superconducting gapping of the FS. The dielectric gapping on the d sections may occur due to either the electron-hole (Coulomb) interaction (the excitonic insulator)[97,98] or the electron-phonon one (the Peierls insulator).[99] The superconducting gapping is a result of the conventional BCS electron pairing and spans the whole FS. Had the constant of interaction that is responsible for the dielectric gapping of the FS been switched off, one would have obtained a "parent" BCS superconductor, characterized by the "bare" zero-T order parameter Δ_0^*; i.e., the corresponding critical temperature (fictitious) would have been $T_c^*=\frac{\gamma}{\pi}\Delta_0^*$ ($\gamma=1.7810\ldots$ is the Euler constant, and the Boltzmann constant $k_B=1$) and within the interval $0\leqslant T\leqslant T_c^*$ its OP Δ^* would have varied following the Mühlschlegel dependence $\Delta^*(T)=\Delta_0^* M\ddot{u}(T/T_c^*)$, with $M\ddot{u}(0)=1$.[100] Accordingly, a uniform gap $\Delta^*(T)$ would have developed on the whole FS within this T interval. On the other hand, if the constant of interaction that is responsible for the superconducting pairing had been switched off, we would have had, at $T<T_d^*$, a parent CDW-metal (CDWM) phase with the dielectric OP $\tilde{\Sigma}^* =\Sigma^* e^{i\varphi}$, characterized by the amplitude Σ^* and the phase φ. The phase φ does not depend on T, is fixed by various factors both in the excitonic[101] and Peierls[99,102] scenarios, and acquires the value either 0 or π in the first case or an arbitrary value in the Peierls state. The dielectric OP amplitude is $\Sigma^*(T)=\Sigma_0^* M\ddot{u}(T/T_d^*)$, with the zero-$T$ amplitude $\Sigma_0^*=\frac{\pi}{\gamma}T_d^*$. Then, a uniform gap $\Sigma^*(T)$ would have developed on the d sections of the FS, the n section remaining ungapped (undielectrized). Of course, the SOP Δ^* should also be characterized by a certain phase, but the latter does not enter into bulk thermodynamic and transport characteristics and can be put equal to zero from the very beginning. Neglecting the superconducting OP phase means that the Josephson currents in junctions involving CDWS's[103] are eliminated from consideration here from the outset. The parameters μ, Δ_0^*, Σ_0^*, and φ comprise a complete set of "bare" parameters to describe the CDWS.

In the framework of the self-consistent theory of the partially gapped CDWS, the superconducting and dielectric gappings can coexist only if the relationship

$$\Sigma_0^* > \Delta_0^* \tag{4}$$

is satisfied.[78] Hence, as the temperature becomes lower, the CDWS undergoes first the dielectric (structural) phase transition at the actual T_d, which therefore coincides with T_d^* ($T_d^*=T_d$). As the temperature continues to decrease, the CDWS behaves as its CDWM parent phase; i.e., the OP

$$\tilde{\Sigma}(T) = \tilde{\Sigma}_0^* \text{Mü}(T/T_d) \tag{5}$$

and the corresponding dielectric gap $|\tilde{\Sigma}(T)|$ on the d FS sections develop, until the actual superconducting critical temperature $T_c < T_d$ is reached. From this point of view, we may assert that superconductivity emerges in a dielectrically gapped CDWM, and, owing to condition (4), the dielectric gapping must be partial for the superconducting state to have a chance to appear. Here, we shall not deal with the more subtle problems of the coexistence between competing superconducting and excitonic (Peierls) pairings due to either an additional doping of the initial degenerate semimetallic state[98,104] or a substantial difference of the cutting factors in the self-consistency equations for both kinds of order parameters.[105,106]

Since, in our case, quasiparticles at the nested sections of the FS already participate in CDW pairing, their ability to take part in the superconducting one is reduced, so that the actual T_c does not coincide with T_c^* of the superconducting parent phase. Within the interval $0 < T < T_c$, the resulting self-consistent OP Δ behaves as

$$\Delta(T < T_c) = \Delta_0 \text{Mü}(T/T_c), \tag{6}$$

with

$$\Delta_0 = (\Delta_0^* \Sigma_0^{*-\mu})^{1/(1-\mu)} < \Delta_0^*, \tag{7}$$

which, in its turn, determines the observable superconducting critical temperature, $T_c = \frac{\gamma}{\pi}\Delta_0$, and the gap $\Delta(T)$ on the n FS section. The gap $\Delta(T)$ is unique on the whole FS due to the strong mixing of the pairing interaction matrix elements.[79,107]

Within the same temperature interval, the CDW-induced quasiparticle properties are also modified by the emergence of Δ in comparison to those in the parent CDWM phase. As a result, the dielectric OP $\Sigma(T)$ behaves at $T < T_c$ not similar to dependence (5) but in a rather nontrivial manner. Namely, the "combined" uniform gap

$$D(T) = \sqrt{\Delta^2(T) + \Sigma^2(T)} \tag{8}$$

appears on the d FS sections and behaves identically to Eq. (5),

$$D(T < T_c) = \Sigma_0^* \text{Mü}(T/T_d). \tag{9}$$

It means that Eq. (9) comprises a smooth continuation of the "purely dielectric" $\Sigma(T)$ gap existing in the range $T_c < T < T_d$ into the $T < T_c$ region. The $\Sigma(T)$ dependence at $T < T_c$ is determined by Eqs. (6), (8), and (9), and its nontrivial behavior consists in a monotonous decrease as T tends to zero. If T_c and T_d are close enough for a definite set of parameters $(\mu, \Delta_0^*, \Sigma_0^*)$, the DOP Σ can even become smaller than the SOP Δ, in spite of the $T_c < T_d$ inequality, whereas the gap D is always larger than the gap Δ. The parameters μ, $\Delta_0 = \Delta(T=0)$, and $D_0 = D(T=0)$, as well as the phase φ, constitute a complete "experimental" set of parameters, from which the "theoretical" set $(\mu, \Delta_0^*, \Sigma_0^*, \varphi)$ can be readily deduced and vice versa.

In what follows, the Green's functions (GF's) of the CDWS will be used as input quantities. To calculate quasiparticle tunnel currents only three of them are necessary:[16]

$$\mathcal{G}_n^{\text{CDWS}}(\mathbf{p}, \omega_n) = -\frac{i\omega_n + \xi_3(\mathbf{p})}{\omega_n^2 + \xi_3^2(\mathbf{p}) + \Delta^2}, \tag{10}$$

$$\mathcal{G}_d^{\text{CDWS}}(\mathbf{p}, \omega_n) = -\frac{i\omega_n + \xi_1(\mathbf{p})}{\omega_n^2 + \xi_1^2(\mathbf{p}) + D^2}, \tag{11}$$

$$\mathcal{G}_c^{\text{CDWS}}(\mathbf{p}, \omega_n) = -\frac{\tilde{\Sigma}}{\omega_n^2 + \xi_1^2(\mathbf{p}) + D^2}, \tag{12}$$

where $\omega_n = (2n+1)\pi T$, $n = 0, \pm 1, \pm 2, \ldots$, which can be derived following the usual technique.[108] The first and second GF's (n and d) correspond to the quasiparticle propagation from the relevant FS section: n or d. The Green's function $\mathcal{G}_c^{\text{CDWS}}$ corresponds to the electron-hole interaction between quasiparticles from different d FS sections. It describes the CDW pairing and to a certain extent is analogous to the Gor'kov Green's function $\mathcal{F}$ of BCS superconductors.[100]

B. Quasiparticle tunnel current

First, it is necessary to comment on the isotropy of the problem. Both Δ and Σ are assumed s-wave symmetrical OP's. Nevertheless, the FS is actually anisotropic, since relation (2) implies effective deviations from its sphericity at least at the d sections. The degree of FS distortion is just described by the parameter μ. Therefore, generally speaking, the tunneling should be directional.[109–112] However, in our phenomenological approach we assume all matrix elements of the tunnel Hamiltonian[113] equal, so that a complete loss of directionality takes place. Additionally, the loss of the CVC directionality may be caused by the presumed nonhomogeneous patch structure of the CDWS (see below).

The quasiparticle tunnel current J through the SIN junction between a homogeneous CDWS and a normal metal is calculated according to the Larkin-Ovchinnikov approach.[114] In our case, it is a sum of three terms J_i,

$$J(V) = \sum_{i=n,d,c} J_i(V), \tag{13}$$

of the same structure

$$J_i \propto \frac{1}{R} \operatorname{Re} \int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\omega \frac{\operatorname{Im} G_i^{\text{CDWS}}(\omega') G^{\text{N}}(\omega)}{\omega' - \omega + eV + i0}. \tag{14}$$

Here, R is the tunnel resistance of the junction in the normal state and $V \equiv V_{\text{N}} - V_{\text{CDWS}}$ is the bias voltage across the junction reckoned from the potential of the CDWS electrode. Thus, each component of the tunnel current is a function of the product of two temporal Green's functions: the normal-metal one $G^{\text{N}}(\omega)$ and that for the CDWS, $G_i^{\text{CDWS}}(\omega)$. The latter can be obtained from the corresponding temperature Green's function of the CDWS (10)–(12) by a well-known procedure.[103,114] The explicit expressions for the current components are

$$J_n = \frac{(1-\mu)}{4eR} \int_{-\infty}^{\infty} d\omega K(\omega, V, T) |\omega| f(\omega, \Delta), \tag{15}$$

$$J_d = \frac{\mu}{4eR}\int_{-\infty}^{\infty} d\omega K(\omega,V,T)|\omega| f(\omega,D), \tag{16}$$

and

$$J_c = \frac{\mu\Sigma\cos\varphi}{4eR}\int_{-\infty}^{\infty} d\omega K(\omega,V,T)\operatorname{sgn}(\omega) f(\omega,D), \tag{17}$$

where

$$K(\omega,V,T) = \tanh\frac{\omega}{2T} - \tanh\frac{\omega - eV}{2T} \tag{18}$$

and

$$f(\omega,x) = \frac{\theta(|\omega| - x)}{\sqrt{\omega^2 - x^2}}. \tag{19}$$

One should note that the coexistence of two kinds of gaps in the cuprate tunnel spectra has also been taken into account in a different way.[115] Namely, those gaps were regarded as coherent and incoherent ones. According to the latter scenario, spatial modulations of the coherent gap reveal themselves[116] as the checkerboard patterns observed in the tunnel spectra of BSCCO.[61,63,67,117–119]

The feature points of the CVC for the quasiparticle current (13) through the CDWS-I-N junction are located at biases $eV = \pm\Delta$ and $\pm D$. The main distinction from the CVC in the BCS-superconductor-I-N case is the absence of CVC symmetricity with respect to the inversion of the bias voltage sign (it is exactly what is required by the experiment). Since there are three Green's functions of the CDWS (10)–(12), the total current (13) the CDWS-I-N junction is composed of three components: J_n, J_d, and J_c (against only one component in the BCS-I-N case). One can easily check that the symmetricity of the currents J_d, Eq. (16), and J_n, Eq. (15), is the same as for tunnel junctions with normal metals and superconductors,

$$J_{d,n}(-V) = -J_{d,n}(V), \tag{20}$$

while

$$J_c(-V) = J_c(V). \tag{21}$$

The anomalous behavior of the J_c current component is directly connected to the dependence of the Green's function (12) on Σ rather than on Σ^2. The summation of two antisymmetric components (20) and a symmetric one (21) makes the total quasiparticle current and the corresponding quasiparticle conductance nonsymmetric.

A more detailed analysis[16] shows that the main role in determining the $J(V)$ behavior belongs to the components J_d and J_c, because their logarithmic singularities, as is seen from Eqs. (11) and (12), are located exactly at $eV = \pm D$ and the character of their asymptotes at $eV \to \pm D$ is very similar, while the symmetrical properties (20) and (21) are different. Therefore, the CVC peculiarities of the J_d and J_c components at $eV = \pm D$ can either enhance or compensate each other. In the latter case, the logarithmic singularity can even be transformed into a cusp (in the limit $\Delta \to 0$). The degree of mutual enhancement or compensation depends on the value of φ. Moreover, the degree μ of the dielectric FS gapping affects the relationship between the components J_d and J_c, on the one hand, and the component J_n, on the other hand [see the preintegral factors in Eqs. (15)–(17)]. Such a nonsymmetricity of the $G(V)$ dependence for tunnel junctions involving CDWSs and the degree of its manifestation, which, in principle, is controlled by all the model parameters, constitutes one of the key points of the proposed theory.

It should be noted that CDW-driven gaps should manifest various features, which were described above, in the absence of superconductivity as well.[120] This has been clearly demonstrated in recent tunnel measurements of stacked structures of the Peierls insulator $NbSe_3$ (the symmetric junctions CDW-I-CDW).[121]

C. CDWS inhomogeneity

The second key point of our theory is connected with the intrinsic nonhomogeneity of the CDWS state in high-T_c cuprates, which is inherent even to the best BSCCO single-crystal specimens. First of all, we would like to point to regular modulated structures of charge distribution in BSCCO, including stripelike and checkerboard ones, which were found in a number of experiments (see, e.g., Refs. 53–62 and 65–68). Such modulated structures remarkably resemble CDW's, and their appearance even correlates with that of the pseudogap.[61] A two-dimensional DOS modulation with an approximately five-lattice-constant periodicity was also found by scanning tunnel microscopy (STM) in the cuprate $Bi_2Sr_{1.6}La_{0.4}CuO_{6+\delta}$ with $T_c \approx 34$ K.[122] In $La_{2-x}Sr_xCuO_4$, inelastic neutron-scattering measurements showed that a dynamic charge inhomogeneity of the stripe-order type reveals itself as a strong bond-stretching phonon anomaly.[68] The dynamical $4a_0 \times 4a_0$ charge order was claimed to be uncovered in BSCCO and, quite reasonably, was considered as a possible candidate for the hidden order in the pseudogap regime of pure bulk crystals.[66] It should be indicated that although the existence of a CDW-like ordering in BSCCO is supported by the analysis of the experimental data,[59] it is not clear yet whether CDW's in this material are unidirectional (stripes) or bidirectional (checkerboard) ones.[123] The uncertainty is due to the quenched impurity potential.[96,123,124]

Indeed, it is well known that the superconducting gap and pseudogap values are distributed over the samples' surfaces of high-T_c oxides in a patchlike irregular manner. We insist on our interpretation that small energy gaps with large and narrow coherence peaks and larger gaps with lower and broadened peaks have a different nature and are generated by superconducting and CDW OP's, although many authors consider both types of gaps as associated with superconductivity.[124] The evidence for such inhomogeneities in the BSCCO and related systems is rather strong and was obtained by various methods (see, e.g., Refs. 19, 28, 59, 60, 65, 66, 68, and 125–138). In particular, STM measurements of $Bi_{2-x}Pb_xSr_2CuO_y$ oxides demonstrated separation in specimens with the SOP Δ ranging within the limits of 13-30 meV over a (12.5×12.5)-nm^2[22] area.[132] At the same time, the dependence $G(V)$ measured for some areas exhibited a pseudogaplike behavior with $\Pi \leqslant 30$ meV. It should be noted that the gap histogram plotted in the work[132] did not make any distinction between Δ and Π. A similar picture was observed for almost optimally doped BSCCO specimens:[133] STM spectra revealed 30-Å spots with "low" (25-30 meV) and "high" (50-75 meV) values of the gaps. In a trilayer material $TlBa_2Ca_2Cu_2O_{10-\delta}$, a wide spread of apparent Δ from 12 to 71 meV exists.[10] Among the gaps, these authors separate out smaller ones ranging from 12 to 21 meV (coined by them as gap I) and larger ones ranging from 35 to 71 meV (gap II). There are junctions with CVC's showing only gap I. There are junctions with CVC's demonstrating only gap II. Finally, there are junctions with two gaplike features with faint signatures of the smaller gap at about 20 meV. This selectivity may be partly due to the directionality of tunneling.

Nanoscale electronic disorder of superconducting properties in optimally doped single-layer $Bi_2Sr_{1.6}L_{0.4}CuO_{6+\delta}$ (L =La or Gd), as well as in BSCCO single crystals, was also revealed by STM measurements.[137] As distinct from the results of Refs. 10 and 133, the gap histogram for BSCCO[137] may be rather considered as a wide distribution around a single mean value. At the same time, histograms for $Bi_2Sr_{1.6}La_{0.4}CuO_{6+\delta}$ and, especially, $Bi_2Sr_{1.6}Gd_{0.4}CuO_{6+\delta}$ can be regarded as two-gap patterns.[137]

Heterogeneity in all listed cases was observed owing to the short coherence length of oxide superconductors, $\xi_0 \approx 15-20$ Å. Because of proximity effects, the obtained spectra constitute a structure which is already averaged to a certain extent, so that even for STM spectra centered around two mean gap values neither small nor large gaps could be unequivocally identified with Δ and Π (in our interpretation, Δ and Σ). It is worth mentioning that in BSCCO, the peak heights at the gap edges are larger for patches with smaller gap values.[133] More direct evidence for the gap-pseudogap interplay was found in interlayer tunneling studies of BSCCO mesa structures.[28] Namely, both features coexisted in the same $G(V)$ dependences resembling earlier results.[25,26] We stress that the employment of the short-pulse technique in Ref. 28 rules out the overheating origin of the corresponding double-gap spectra. This problem was first raised for $BaPb_{1-x}Bi_xO_3$ ceramics[139,140] and later discussed for BSCCO mesas[141–145] and semimetallic $SrPbO_{3-\delta}$ with partially localized charge carriers.[146]

One sees that BSCCO and other Bi-based oxides seem to be rather unstable against some types of irregular phase separation, in contrast with, e.g., $YBa_2Cu_3O_{6+y}$.[147] The relative stability of the latter substance is associated with a much higher uniformity of the hole doping distribution in comparison to that in the BSCCO system.

It is remarkable that an inherent inhomogeneity was also indirectly observed in such copperless oxides as $BaPb_{1-x}Bi_xO_3$ (Refs. 139 and 140) and $Ba_{1-x}K_xBiO_3$.[148]

The microscopic nature of the random nonhomogeneities in BSCCO and other hole-doped cuprates is not fully understood. Nevertheless, apparent positive correlations between dopant positions and gap peak heights were observed.[127,129] Since oxides are disordered nonstoichiometric compounds,[149] inhomogeneity of their electronic properties is inevitable. For instance, electronic DOS inhomogeneities induced by doping-induced disorder were detected by STM and photoemission measurements in single crystals of the nonsuperconducting $Ca_{2-x}Sr_xRuO_4$ with $0.1 \leqslant x \leqslant 2.0$.[150]

The gap-dopant position correlations were explained[151] at the semiphenomenological level by introducing random modulations of the local pair potential in the site-dependent BCS theory. It was also found in photoemission measurements[152] that every oxygen dopant induces a small and broad nondispersive peak at -0.8 eV and the dopant states hybridize with those of Cu atoms in the superconducting plane of BSCCO. Notwithstanding a certain agreement between the theory[151] and the STM nonhomogeneity observations,[129] we think that the treatment cannot be fully adequate without invoking CDW's (pseudogaps).

The correlation between the superconducting gap at the FS and the strength of the coupling constant[151] was confirmed in Ref. 153, and, moreover, in agreement with the experiments,[65] an *anticorrelation* between $\Delta(0)$ and the relevant boson mode energy, responsible for superconductivity, was found in the disordered oxide structure (see below).

Until now we have been talking about the first derivative $G(V)$ of the quasiparticle current $J(V)$. If not to follow the questionable viewpoint[12,32,45–48] that strong-coupling features manifest themselves already in $G(V)$, one should examine the second derivative d^2J/dV^2 to elucidate the microscopic origin of superconductivity. The peculiarities of d^2J/dV^2 must indicate relevant frequencies of boson mediators in the actual Cooper pairing,[50] whatever the pairing boson-exchange mechanism.

As for the latter, we note that, in the case of BSCCO and other cuprates, adopting the d-wave symmetry of the superconducting OP as at least a predominant one[80,84,154–156] led to the conclusion that the underlying mechanism of superconductivity is Coulombic.[155] Therefore, the Hubbard model became extremely popular[155,157–160] and spin-wave resonance excitations observed in many high-T_c oxides[161–166] are offered as a true source of the Cooper pairing there.[45,167,168] Other investigators either advocate more traditional phonons as a glue for paired electrons[169,170] or consider the interplay of both mechanisms.[156] In any case, estimations show that in spite of the importance of the resonant mode in determining the photoemission and optical properties of cuprates, the smallness of the relevant spin-wave peak spectral weight makes its dominant role as a pairing boson impossible.[171] On the other hand, even for the lattice-vibration-induced superconductivity the latter might have a bipolaronic (Bose-Einstein) character rather than a BCS one.[172] Then, the very shape of the $G(V)$ dependence would be different.[115,173,174]

Recent experiments clearly demonstrated that the phonon scenario is more probable. Specifically, peaks in d^2J/dV^2 seem to be caused by local lattice vibrations, since the substitution of ^{18}O for ^{16}O reduces the average mode energy by about 6%.[65] The idea[175] (see also Ref. 176) of an inelastic (via apical oxygen-atom phonons) tunneling between the topmost CuO_2 layers in BSCCO samples and STM tips qualitatively agrees with the experiments concerned. Nevertheless, boson satellites, according to Ref. 175, should noticeably distort the first derivative $G(V)$ as well.

STM measurements[65] revealed several peculiarities of d^2J/dV^2. The main local phonon-mode energies $\Omega(\mathbf{r})$ are

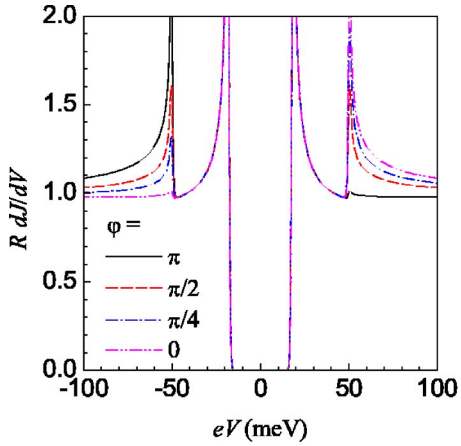

FIG. 1. (Color online) Influence of the parameter φ on the dependence dimensionless differential conductance $RdJ/dV=RG(V)$ of the tunnel junction between a homogeneous charge-density-wave superconductor (CDWS) and a normal metal versus the bias voltage V across the junction expressed in energy units. Here, J is the quasiparticle tunnel current, R is the resistance of the junction in the normal state, and e is the elementary charge. The bare CDWS parameters are $\Delta_0^*=20$ meV, $\Sigma_0^*=50$ meV, and $\mu=0.1$; the temperature $T=4.2$ K.

easily identified and rapidly vary with the two-dimensional spatial coordinate $\mathbf{r}$. Its mean value $\bar{\Omega}$ is 52 meV with a statistical spread of ±8 meV. We emphasize that $\bar{\Omega}$ does not coincide with the dip or hump positions and, in our opinion, the DHS has another origin than $\bar{\Omega}$ does. The distributions $\Omega(\mathbf{r})$ and $\Delta(\mathbf{r})$ anticorrelate, whereas the oxygen-dopant locations $O(\mathbf{r})$ positively correlate with $\Delta(\mathbf{r})$. Hence, the intrinsic nonhomogeneity of electron and phonon characteristics has a profound effect on the superconductivity of BSCCO oxide.

III. RESULTS OF CALCULATIONS

As was shown above, the CDWS can be characterized by either of the parameter sets $(\mu,\Delta_0^*,\Sigma_0^*,\varphi)$ or $(\mu,\Delta_0,D_0,\varphi)$. In calculations below, the former one will be used as more convenient to treat the CDWS nonhomogeneity.

Generally speaking, each of the model parameters may have a certain dispersion, and all four can be scattered concurrently, in accordance to what happens in cuprates. But it would be very difficult to analyze the results of such calculations. Therefore, we shall consider the variation of each parameter independently; we shall also do it to trace the tendency which each such variation might bring about. The basic formulas which are needed for this analysis are Eqs. (6), (7), and (9). The main reason is that it is the gaps Δ and D that determine the positions of CVC singularities $eV=\pm\Delta$ and $\pm D$ in the dispersionless case.

First, we consider the phase φ. This parameter does not enter the set of equations listed above and, hence, does not influence the singularity positions. Its role was discussed in detail in Sec. II B. Figure 1 illustrates this role in the case of an SIN junction, which includes a CDWS electrode with dispersionless parameters Δ_0^*, Σ_0^*, and μ, the specific values of the latter being chosen in accordance with the experimental data for the BSCCO-I-N junction.[2] If the voltage is reckoned from the potential of the CDWS electrode, the value $\varphi=0$ results in the largest possible mutual enhancement of the J_d and J_c peculiarities at the bias $eV=+D$ and their largest possible compensation at $eV=-D$. On the contrary, if $\varphi=\pi$, the largest enhancement of the resulting singularity occurs at $eV=-D$ and the largest compensation at $eV=+D$. If $\varphi=\pi/2$—i.e., $\cos\varphi=0$—the antisymmetric component J_c vanishes, so that the CVC becomes symmetric. Since the parameters of the CDWS electrode are constant over the bulk, the singularities are slightly smeared only by thermal effects at $T\neq 0$. The dependence for $\cos\varphi=-1$ will serve (see explanations below) as a reference for one of our subsequent calculations, which accounts for the dispersion of the CDWS parameters. But while comparing the amplitudes of the singularities, it is evident that, e.g., in the case $\varphi=0$, the dispersion concerned would maximally affect the CDW-driven peculiarity at $eV=D$. Any spatial spread of the phase φ would result in the reduction of the averaged $|\cos\varphi|$ and the transformation of the CVC to a more symmetrical pattern. Some experimental results do demonstrate almost symmetrical CVC's for nonsymmetric SIN junctions (see, e.g., Ref. 12). As is clear from the discussion above, in the framework of our theory it can be achieved in two ways: if either the value of φ in the CDWS electrode is close to $\pi/2$ or the averaging over φ occurs.

The majority of nonsymmetric CVC's for SIN junctions reveal the DHS only in its negative-voltage branch[2–4], thus, the occupied CDWS electron states below the Fermi level are probed. In our approach, it corresponds to the phase φ close to π. This phenomenon may be associated with some unidentified features of the CDW behavior near the sample surface. The explanation of this fact can be done only at the microscopic level, which is beyond the scope of our study. Therefore, in what follows, we confine our calculations to the case $\varphi=\pi$. Now, the spatially nonhomogeneous CDWS electrode can be described by the dispersion of any of the remaining three parameters $(\mu,\Delta_0^*,\Sigma_0^*)$. Each parameter x was regarded as independently distributed within the interval $[x_0-\sigma,x_0+\sigma]$ around the mean value x_0; the normalized weight function

$$W(x)=\frac{15}{16\sigma^5}[(x-x_0)^2-\sigma^2]^2 \quad (22)$$

within the interval $[x_0-\sigma,x_0+\sigma]$ and equal to zero beyond it was used. This polynomial ensures a smooth vanishing of the function $W(x)$ together with its first derivative outside the indicated interval. As a result, it leads to smoother $G(V)$ than the Gaussian approximation used in Refs. 153 and 177 when analyzing the effects of SOP inhomogeneity in cuprates. In principle, the specific form of the weight function $W(x)$ should be either derived from a microscopic theory or taken from the experiment. However, since our approach is phenomenological, the function $W(x)$ can be regarded as a model one. At the same time, it is obvious that the replacement of function (22) by any other, which is centered around

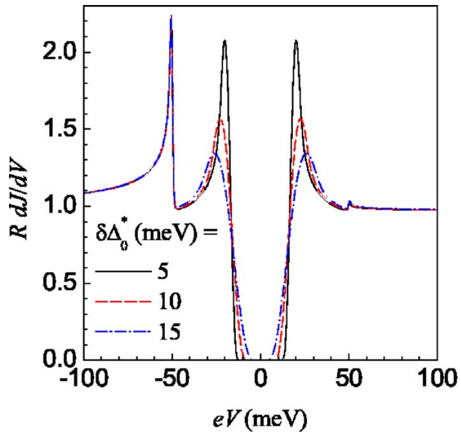

FIG. 2. (Color online) Dependences $G(V)$ for the same junction as in Fig. 1 (the case $\varphi=\pi$) but for inhomogeneous CDWS with various dispersions $\delta\Delta_0^*$.

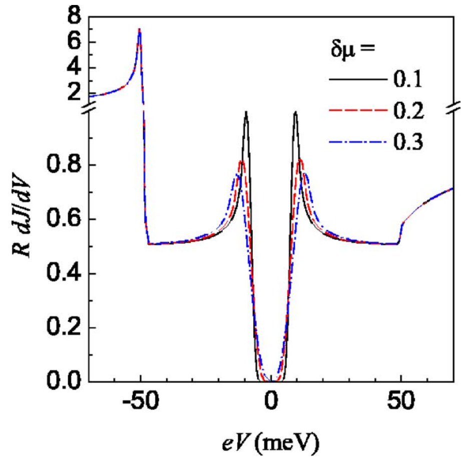

FIG. 3. (Color online) The same as in Fig. 2 but for $\Delta_0^*=20$ meV, $\Sigma_0^*=50$ meV, and various dispersions $\delta\mu$ centered around $\mu=0.5$.

x_0, could lead only to numerical differences in the final result of an order of a few percent, leaving the basic conclusion of the work unchanged.

We note that our method as well as the related approaches of Refs. 153 and 177 deals with the averaging of the OP's themselves, whereas one may alternatively start from the mean-field Hamiltonian with a lattice-site impurity disorder.[178] The independent averaging over different parameters of the problem can be justified if one takes into account that Δ and Σ are induced by different combinations of bare coupling constants.[79,106,179]

A similar assumption was made in Ref. 153 while treating spatially distributed coupling constants and boson frequencies in nonhomogeneous cuprates. Other phenomenological approaches to the problem of the cuprate nonhomogeneity were applied there as well. Specifically, a random impurity field,[96] a Hamiltonian with a lattice-site impurity disorder and two sorts of randomly distributed cells with different gap parameters,[124] and a quenched finite-range impurity potential[123] were introduced.

Consider now the dispersion of the bare SOP Δ_0^*. According to our basic equation set, the scatter of Δ_0^* can influence only the position of the $eV=\pm\Delta$ peculiarities, leaving the $eV=\pm D$ ones untouched. Figure 2 illustrates the corresponding $G(V)$ curves for various scattering amplitudes [the parameter σ in Eq. (22)]. It is evident that at higher dispersions of Δ_0^* the profile of $G(V)$ in the intragap region approaches the V-like experimental data, which are usually considered as the clear-cut consequence of the $d_{x^2-y^2}$ symmetry of the superconducting order parameter or at least an extended s-wave symmetry (see the classification of the pairing states in Refs. 180–182). The V-like intragap asymptotics of $G(V)$ is of no surprise, because, from the mathematical point of view, averaging over the spatial distribution of the gap would give rise to results similar to those obtained when averaging over the SOP angle in the **k** space. At the same time, the D-driven singularities remained almost intact and no structures similar to the dip-hump are observed. Therefore, the dispersion of the Δ_0^* parameter alone turns out insufficient to explain the experimental data. It should be pointed out that, if the same kind of calculations had been made, say, for an extended s-wave SOP, one would have required a smaller or no dispersion $\delta\Delta_0^*$ to reach the same intragap filling as in Fig. 2, because the SOP nodes themselves, if any, might have substantially contributed to the V-like character of $G(V)$ at $V<\Delta$.

A similar situation occurs for the parameter μ. Really, the variation of this parameter influences only the position of the $eV=\pm\Delta$ peculiarities, leaving the $eV=\pm D$ ones fixed, as it was in the previous case. Moreover, the resulting CVC's for those μ values, which are typical of CDWS's (0.05-0.2), turn out indistinguishable for a naked eye. For this reason, in Fig. 3, we illustrate the role of the parameter μ dispersion using a very high value 0.5 for the parameter x_0 in function (22). As can be deduced from a comparison of Figs. 2 and 3, the roles of the spread of the parameters Δ_0^* and μ are analogous. Those two figures (compare the relationship between the amplitudes of the $\pm\Delta$ and $\pm D$ singularities in them) also give evidence for another role of the parameter μ—namely, that it governs the amplitudes of the quasiparticle current components [see Eqs. (15)–(17)]—so that the proper selection of μ may diminish the J_d and J_c currents to such values when even a small dispersion of another parameter may totally smear the $eV=-D$ singularity to the DHS. But such values of μ would be very small, thus being of academic interest only. Hence, the spread of the parameter μ, similarly to that of Δ_0^*, cannot explain alone the emergence of the DHS. At the same time, μ effectively governs its amplitude (see below).

The results of calculations resemble the observed $G(V)$ dependences for BSCCO much more if it is the parameter Σ_0^* that is assumed to scatter (Fig. 4). Indeed, according to our basic equations, all four CVC peculiarities $eV=\pm\Delta$ and $\pm D$ become smeared, although to various extents: the large singularities $eV=\pm\Delta$ almost preserve their shape, the large singularity $eV=-D$ transforms into a DHS, and the small one $eV=D$ disappears on the scale selected. The one-polarity DHS of experimental CVC's (Ref. 2) is reproduced excellently. Owing to relationship (7), the actual parameter Δ also disperses, but due to the small value of μ, this fluctuation becomes too small to be observed in the plot.

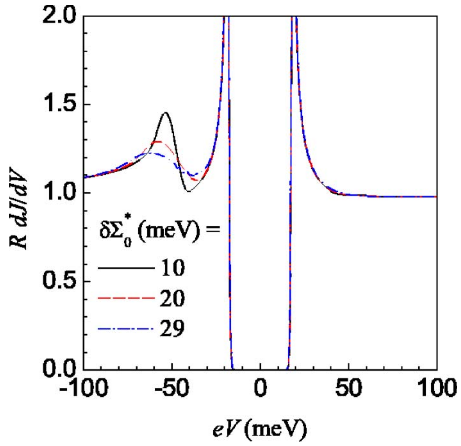

FIG. 4. (Color online) The same as in Fig. 2 but for $\Delta_0^*=20$ meV, $\mu=0.1$, and various dispersions $\delta\Sigma_0^*$ centered around $\Sigma_0^*=50$ meV.

Figure 5 clearly demonstrates the influence of the parameter μ on the amplitude of the DHS. It is evident that μ governs this amplitude very effectively. On the other hand, in order to describe the experiment adequately it is sufficient to admit that only a small fraction (of about 5%-20%) of the FS is nested and distorted by the dielectric gap. For comparison, $\mu\approx 0.2$ in CDW metal $NbSe_3$ (Ref. 183) and $\mu\approx 0.1$ in Cr-Re alloys where the spin density wave is observed.[184]

Roughly speaking, the parameters Δ_0^* and Σ_0^* are responsible for the locations of peculiarities (the coherent peaks and the DHS) in the CVC's and the parameter μ for the amplitudes of the DHS. Thereby, the observed correlation (not a *direct proportionality*) between a DHS energy and T_c (Refs. 12 and 34) is explained by our theory.

Finally, for completeness, the role of T in our scenario of the DHS phenomenon is shown in Fig. 6. As could be expected, thermal smearing and spatial averaging act synergetically to suppress the CDW influence on the tunnel conductance. Therefore, although, in principle, the DHS should preserve above T_c in the pseudogap state, which we consider a partially gapped state with CDW's, its observation may be

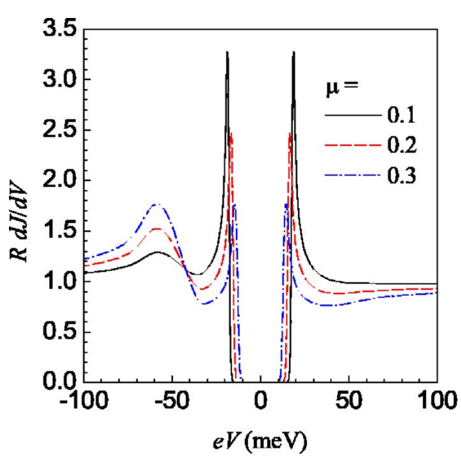

FIG. 5. (Color online) Influence of the parameter μ on the dip-hump structure amplitude for $\Delta_0^*=20$ meV, $\Sigma_0^*=50$ meV, $\delta\Sigma_0^*=20$ meV, $\varphi=\pi$, and $T=4.2$ K.

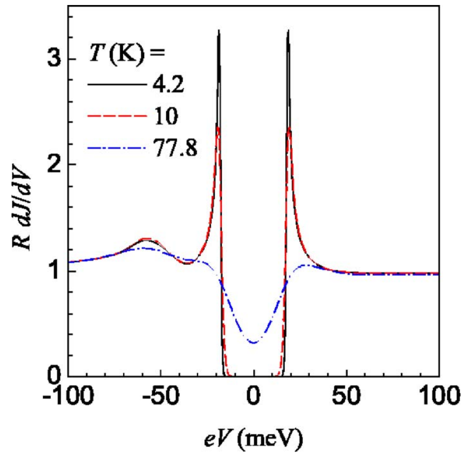

FIG. 6. (Color online) Influence of the temperature T on the $G(V)$ shape for $\Delta_0^*=20$ meV, $\Sigma_0^*=50$ meV, $\delta\Sigma_0^*=20$ meV, $\mu=0.1$, and $\varphi=\pi$.

obscured. In the competitive scenario of the localized vibration (spin) mode-induced DHS, satellites at $eV=\Delta+\omega_0$ must disappear above T_c.[51]

Thus, the theory proposed can describe, at least qualitatively, the arising DHS-induced asymmetry in the CVC's of nonsymmetric SIN tunnel junctions. It should be emphasized once more that in a real inhomogeneous CDWS electrode the values of all three (Δ_0^*, Σ_0^*, and μ) or even four (including φ) parameters can be characterized by simultaneous scattering over the volume. We have reported the results of calculations executed with the assumption that only one of those parameters varies. Therefore, one could hardly expect an exact quantitative coincidence of theoretical and experimental data. At the same time, the experimental data (see, e.g., Refs. 2, 4, 7, 12, 95, 133, 185, and 186) undoubtedly include a background component of the unknown nature, which our theory obviously does not cover. Nevertheless, the position and the amplitude of the DHS in the CVC's are reproduced excellently.

In particular, to illustrate the theory, the kit of the bare parameters ($\mu=0.1$, $\Delta_0^*=20$ meV, and $\Sigma_0^*=50$ meV) was selected in order to approximate a single experimental curve for BSCCO-I-N junction (the 62-K curve in Fig. 1 of Ref. 2), where the manifestation of the DHS is most typical. The shape of the DHS in this curve turned out well simulated, provided the dispersion $\delta\Sigma_0^*=20$ meV (see Fig. 4). Moreover, taking into account the dispersion of each individual problem parameter leads to the evolution of the "reference" CVC shape (which corresponds to some fixed parameter values; see Fig. 1) towards experimentally observed CVC patterns. Once more, as was indicated above, the background of an obscure origin is the main difference, which remains unexplained.

Note that the asymmetry of tunnel CVC, associated with the availability of DHS, is sometimes asserted to be more characteristic of overdoped high-T_c superconductors. However, such asymmetry is also observed for underdoped superconductors.[5] From the viewpoint of the theory proposed, a lower frequency of DHS observation in underdoped superconductors can be associated, in particular, with their higher homogeneity. To confirm the aforesaid, the CVC's of tunnel junctions including $YBa_2Cu_3O_{6+y}$, where the DHS is absent, might be mentioned. In this material holes are distributed much more uniformly over the specimen than in BSCCO.[147]

On the other hand, the DHS-induced CVC asymmetry may be more typical of overdoped oxides of the BSCCO type as a result of the pseudogap (CDW, in our interpretation) and superconducting features coming closer near the top of the superconducting dome in the phase diagram T-x.[82]

IV. CONCLUSIONS

Thus, assuming the CDW origin of the DHS in tunnel spectra of high-T_c oxides, in particular, BSCCO, made it possible to qualitatively describe the CVC's. On the other hand, the widely spread alternative explanation[47,48,187] of the DHS, which is based on the existence of a certain boson mode of an unknown nature with energy Ω, cannot account for the observed nonsymmetrical form of the CVC's. In addition, a wide-range oxygen doping of BSCCO leads to a shift of both the Δ and DHS locations, which are not linked with each other by any simple relationship.[188] At the same time, in the framework of the boson-based approach the DHS should be positioned at $eV=\Delta+\Omega$ in SIN and $eV=2\Delta+\Omega$ in SIS junctions.

In our scenario, the locations of Δ and DHS are also correlated, which is traced while doping the material. Nevertheless, the link between two quantities is rather complicated, because the D-peculiarity position depends on the doping-driven shift of T_d in a twofold nonlinear way: through Σ and Δ. The latter quantity, in its turn, also depends on Σ (see Sec. II A). The above calculations show the inaccuracy of the conclusion[32] that a strong observed dip cannot be described under the assumption of its nonsuperconducting nature. It should be noted that our conclusions, although being obtained assuming an s-wave symmetry of both OP's, are quite general and applicable to the situation when their symmetry is reduced (e.g., for the d-wave symmetry[189] or the extended s-wave symmetry). Of course, the magnitudes of the corresponding singularities will be smaller in this case. Since there are quite a number of noncuprate CDW superconductors,[16] a DHS of the type considered in this work might be found there as well. Their detection would be another argument in favor of our viewpoint concerning high-T_c oxides. Anyway, the solution of the DHS problem for BSCCO and its relatives would comprise an important forward step to overcome the current dichotomous state in the science of cuprate superconductivity: one gap versus two gaps.[30,93,190,191]

ACKNOWLEDGMENTS

The authors are grateful to Kasa im. Józefa Mianowskiego and Fundacja na Rzecz Nauki Polskiej for the financial support of their visits to Warsaw and to Toshikazu Ekino (Faculty of Integrated Arts and Sciences, Hiroshima University, Higashi-Hiroshima, Japan), Maciej Maśka (Institute of Physics, University of Silesia, Katowice, Poland), and Karol Wysokiński (Institute of Physics and Nanotechnology Centre, M. Curie-Skłodowska University, Lublin, Poland) for fruitful discussions.

*Electronic address: gabovich@iop.kiev.ua
†Electronic address: voitenko@iop.kiev.ua
[1] H. Won and K. Maki, Phys. Rev. B **49**, 1397 (1994).
[2] N. Miyakawa, J. F. Zasadzinski, L. Ozyuzer, P. Guptasarma, D. G. Hinks, C. Kendziora, and K. E. Gray, Phys. Rev. Lett. **83**, 1018 (1999).
[3] Ch. Renner, B. Revaz, J.-Y. Genoud, K. Kadowaki, and Ø. Fischer, Phys. Rev. Lett. **80**, 149 (1998).
[4] A. K. Gupta and K.-W. Ng, Phys. Rev. B **58**, R8901 (1998).
[5] J. Zasadzinski, L. Ozyuzer, N. Miyakawa, D. G. Hinks, and K. E. Gray, Physica C **341–348**, 867 (2000).
[6] T. Cren, D. Roditchev, W. Sacks, and J. Klein, Europhys. Lett. **54**, 84 (2001).
[7] M. Kugler, G. Levy de Castro, E. Giannini, A. Piriou, A. A. Manuel, C. Hess, and Ø. Fischer, J. Phys. Chem. Solids **67**, 353 (2006).
[8] N. Miyakawa, K. Tokiwa, S. Mikusu, J. F. Zasadzinski, L. Ozyuzer, T. Ishihara, T. Kaneko, T. Watanabe, and K. E. Gray, Int. J. Mod. Phys. B **17**, 3612 (2003).
[9] L. Ozyuzer, Z. Yusof, J. F. Zasadzinski, T-W. Li, D. G. Hinks, and K. E. Gray, Physica C **320**, 9 (1999).
[10] N. Miyakawa, K. Tokiwa, S. Mikusu, T. Watanabe, A. Iyo, J. F. Zasadzinski, and T. Kaneko, Int. J. Mod. Phys. B **19**, 225 (2005).
[11] J. Y. T. Wei, C. C. Tsuei, P. J. M. van Bentum, Q. Xiong, C. W. Chu, and M. K. Wu, Phys. Rev. B **57**, 3650 (1998).
[12] Y. DeWilde, N. Miyakawa, P. Guptasarma, M. Iavarone, L. Ozyuzer, J. F. Zasadzinski, P. Romano, D. G. Hinks, C. Kendziora, G. W. Crabtree, and K. E. Gray, Phys. Rev. Lett. **80**, 153 (1998).
[13] P. W. Anderson, P. A. Lee, M. Randeria, T. M. Rice, N. Trivedi, and F. C. Zhang, J. Phys.: Condens. Matter **16**, R755 (2004).
[14] P. W. Anderson and N. P. Ong, J. Phys. Chem. Solids **67**, 1 (2006).
[15] G. Deutscher, Rev. Mod. Phys. **77**, 109 (2005).
[16] A. M. Gabovich, A. I. Voitenko, and M. Ausloos, Phys. Rep. **367**, 583 (2002).
[17] A. M. Gabovich, A. I. Voitenko, J. F. Annett, and M. Ausloos, Supercond. Sci. Technol. **14**, R1 (2001).
[18] J-P. Hu and K. Seo, Phys. Rev. B **73**, 094523 (2006).
[19] K. Anagawa, T. Watanabe, and M. Suzuki, Phys. Rev. B **73**, 184512 (2006).
[20] K. Anagawa, Y. Yamada, T. Watanabe, and M. Suzuki, Phys. Rev. B **67**, 214513 (2003).
[21] T. Ekino, Y. Sezaki, S. Hashimoto, and H. Fujii, J. Low Temp. Phys. **117**, 359 (1999).
[22] R. A. Klemm, Physica C **341–348**, 839 (2000).
[23] R. A. Klemm, in *Nonequilibrium Physics at Short Time Scales. Formation of Correlations*, edited by K. Morawetz (Springer-

Verlag, Berlin, 2004), p. 381.
[24] J. L. Tallon and J. W. Loram, Physica C **349**, 53 (2001).
[25] V. M. Krasnov, A. Yurgens, D. Winkler, P. Delsing, and T. Claeson, Phys. Rev. Lett. **84**, 5860 (2000).
[26] V. M. Krasnov, A. E. Kovalev, A. Yurgens, and D. Winkler, Phys. Rev. Lett. **86**, 2657 (2001).
[27] G. Q. Zheng, P. L. Kuhns, A. P. Reyes, B. Liang, and C. T. Lin, Phys. Rev. Lett. **94**, 047006 (2005).
[28] Y. Yamada and M. Suzuki, Phys. Rev. B **74**, 054508 (2006).
[29] J-X. Li, C-Q. Wu, and D-H. Lee, Phys. Rev. B **74**, 184515 (2006).
[30] K. Tanaka, W. S. Lee, D. H. Lu, A. Fujimori, T. Fujii, Risdiana, I. Terasaki, D. J. Scalapino, T. P. Devereaux, Z. Hussain, and Z-X. Shen, Science **314**, 1910 (2006).
[31] J. E. Hirsch, Phys. Rev. B **59**, 11962 (1999).
[32] P. Romano, L. Ozyuzer, Z. Yusof, C. Kurter, and J. F. Zasadzinski, Phys. Rev. B **73**, 092514 (2006).
[33] A. Yurgens, D. Winkler, T. Claeson, S-J. Hwang, and J-H. Choy, Int. J. Mod. Phys. B **13**, 3758 (1999).
[34] J. F. Zasadzinski, L. Ozyuzer, N. Miyakawa, K. E. Gray, D. G. Hinks, and C. Kendziora, Phys. Rev. Lett. **87**, 067005 (2001).
[35] M. Suzuki, T. Watanabe, and A. Matsuda, Phys. Rev. Lett. **82**, 5361 (1999).
[36] M. Suzuki and T. Watanabe, Phys. Rev. Lett. **85**, 4787 (2000).
[37] M. Suzuki, K. Anagawa, M. Lmouchter, and T. Watanabe, Physica C **362**, 164 (2001).
[38] J. Bok and J. Bouvier, Physica C **274**, 1 (1997).
[39] J. Bouvier and J. Bok, Physica C **249**, 117 (1995).
[40] R. S. Markiewicz, J. Phys. Chem. Solids **58**, 1179 (1997).
[41] J. Bouvier and J. Bok, in *The Gap Symmetry and Fluctuations in High Temperature Superconductors*, edited by J. Bok, G. Deutscher, D. Pavuna, and S. A. Wolf (Plenum Press, New York, 1998), p. 37.
[42] G. Litak, A. M. Martin, B. L. Györffy, J. F. Annett, and K. Wysokiñski, Physica C **309**, 257 (1998).
[43] G. Litak, Phys. Status Solidi B **229**, 1427 (2002).
[44] M. Eschrig and M. R. Norman, Phys. Rev. Lett. **85**, 3261 (2000).
[45] Ar. Abanov, A. V. Chubukov, and J. Schmalian, J. Electron Spectrosc. Relat. Phenom. **117–118**, 129 (2001).
[46] J. F. Zasadzinski, L. Coffey, P. Romano, and Z. Yusof, Phys. Rev. B **68**, 180504(R) (2003).
[47] J. F. Zasadzinski, L. Ozyuzer, L. Coffey, K. E. Gray, D. G. Hinks, and C. Kendziora, Phys. Rev. Lett. **96**, 017004 (2006).
[48] J-X. Zhu, Jun Sun, Qimice Si, and A. V. Balatsky, Phys. Rev. Lett. **92**, 017002 (2004).
[49] W. Sacks, T. Cren, D. Roditchev, and B. Doucot, Phys. Rev. B **74**, 174517 (2006).
[50] W. L. McMillan and J. M. Rowell, in *Superconductivity*, edited by R. D. Parks (Dekker, New York, 1969), Vol. 1, Chap. 11, p. 561.
[51] A. V. Balatsky, Ar. Abanov, and J-X. Zhu, Phys. Rev. B **68**, 214506 (2003).
[52] D. K. Morr and R. H. Nyberg, Phys. Rev. B **68**, 060505(R) (2003).
[53] A. Bianconi, M. Lusignoli, N. L. Saini, P. Bordet, A. Kvick, and P. G. Radaelli, Phys. Rev. B **54**, 4310 (1996).
[54] J. P. Castellan, B. D. Gaulin, H. A. Dabkowska, A. Nabialek, G. Gu, X. Liu, and Z. Islam, Phys. Rev. B **73**, 174505 (2006).
[55] E. V. L. de Mello and E. S. Caixeiro, J. Supercond. **18**, 653 (2005).
[56] K. McElroy, D-H. Lee, J. E. Hoffman, K. M. Lang, J. Lee, E. W. Hudson, H. Eisaki, S. Uchida, and J. C. Davis, Phys. Rev. Lett. **94**, 197005 (2005).
[57] J. Etrillard, P. Bourges, and C. T. Lin, Phys. Rev. B **62**, 150 (2000).
[58] G. Kinoda, T. Hasegawa, S. Nakao, T. H. K. Kitazawa, K. Shimizu, J. Shimoyama, and K. Kishio, Appl. Phys. Lett. **83**, 1178 (2003).
[59] S. A. Kivelson, I. P. Bindloss, E. Fradkin, V. Oganesyan, J. M. Tranquada, A. Kapitulnik, and C. Howald, Rev. Mod. Phys. **75**, 1201 (2003).
[60] A. Sugimoto, S. Kashiwaya, H. Eisaki, H. Yamaguchi, K. Oka, H. Kashiwaya, H. Tsuchiura, and Y. Tanaka, Physica C **426–431**, 390 (2005).
[61] M. Vershinin, S. Misra, S. Ono, Y. A. Y. Ando, and A. Yazdani, Science **303**, 1995 (2004).
[62] V. Kresin, Yu. Ovchinnikov, and S. Wolf, Phys. Rep. **431**, 231 (2006).
[63] J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, H. Eisaki, S. Uchida, and J. C. Davis, Science **295**, 466 (2002).
[64] N. Momono, A. Hashimoto, Y. Kobatake, S. Bakamura, M. Oda, and M. Ido, Int. J. Mod. Phys. B **19**, 231 (2005).
[65] J. Lee, K. Fujita, K. McElroy, J. A. Slezak, M. Wang, Y. Aiura, H. Bando, M. Ishikado, T. Masui, J-X. Zhu, A. V. Balatsky, H. Eisaki, S. Uchida, and J. C. Davis, Nature (London) **442**, 546 (2006).
[66] A. Hashimoto, N. Momono, M. Oda, and M. Ido, Phys. Rev. B **74**, 064508 (2006).
[67] K. M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, and Z-X. Shen, Science **307**, 901 (2005).
[68] D. Reznik, L. Pintschovius, M. Ito, S. Iikubo, M. Sato, H. Goka, M. Fujita, K. Yamada, G. D. Gu, and J. M. Tranquada, Nature (London) **440**, 1170 (2006).
[69] R. Micnas, J. Ranninger, and S. Robaszkiewicz, Rev. Mod. Phys. **62**, 113 (1990).
[70] P. Miller, B. Janko, and B. L. Györffy, Physica C **210**, 343 (1993).
[71] I. Eremin, M. Eremin, S. Varlamov, D. Brinkmann, M. Mali, and J. Roos, Phys. Rev. B **56**, 11305 (1997).
[72] M. V. Eremin and I. A. Larionov, Pis'ma Zh. Eksp. Teor. Fiz. **68**, 583 (1998).
[73] Y. Zhang, E. Demler, and S. Sachdev, Phys. Rev. B **66**, 094501 (2002).
[74] S. A. Kivelson, Nat. Mater. **5**, 343 (2006).
[75] E. L. Nagaev, Usp. Fiz. Nauk **165**, 529 (1995).
[76] H. D. Yang, P. Klavins, and R. N. Shelton, Phys. Rev. B **43**, 7681 (1991).
[77] E. Morosan, H. W. Zandbergen, B. S. Dennis, J. W. G. Bos, Y. Onose, T. Klimczuk, A. P. Ramirez, N. P. Ong, and R. J. Cava, Nat. Phys **2**, 544 (2006).
[78] A. M. Gabovich, M. S. Li, H. Szymczak, and A. I. Voitenko, J. Phys.: Condens. Matter **15**, 2745 (2003).
[79] G. Bilbro and W. L. McMillan, Phys. Rev. B **14**, 1887 (1976).
[80] J. F. Annett, N. D. Goldenfeld, and A. J. Leggett, in *Physical Properties of High Temperature Superconductors V*, edited by D. M. Ginsberg (World Scientific, River Ridge, NJ, 1996), p. 375.
[81] D. Coffey and L. Coffey, Phys. Rev. Lett. **70**, 1529 (1993).
[82] A. Damascelli, Z. Hussain, and Z-X. Shen, Rev. Mod. Phys. **75**, 473 (2003).

[83] C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. **72**, 969 (2000).
[84] H. Hilgenkamp and J. Mannhart, Rev. Mod. Phys. **74**, 485 (2002).
[85] J. R. Kirtley, C. C. Tsuei, A. Ariando, C. J. M. Verwijs, S. Harkema, and H. Hilgenkamp, Nat. Phys. **2**, 190 (2006).
[86] G. B. Arnold and R. A. Klemm, Philos. Mag. B **86**, 2811 (2006).
[87] R. A. Klemm, J. Supercond. **18**, 697 (2005).
[88] R. A. Klemm, Philos. Mag. B **85**, 801 (2005).
[89] G-m. Zhao, Philos. Mag. B **84**, 3869 (2004).
[90] D. R. Harshman, W. J. Kossler, X. Wan, A. T. Fiory, A. J. Greer, D. R. Noakes, C. E. Stronach, E. Koster, and J. D. Dow, Phys. Rev. B **69**, 174505 (2004).
[91] A. Bussmann-Holder, R. Khasanov, A. Shengelaya, A. Maisuradze, F. La Mattina, H. Keller, and K. A. Müller, Europhys. Lett. **77**, 27002 (2007).
[92] I. I. Mazin and V. P. Antropov, Physica C **385**, 49 (2003).
[93] A. Millis, Science **314**, 1888 (2006).
[94] A. Kapitulnik, A. Fang, C. Howald, and M. Greven, J. Phys. Chem. Solids **67**, 344 (2006).
[95] B. W. Hoogenboom, C. Berthod, M. Peter, Ø. Fischer, and A. A. Kordyuk, Phys. Rev. B **67**, 224502 (2003).
[96] A. Del Maestro, B. Rosenow, and S. Sachdev, Phys. Rev. B **74**, 024520 (2006).
[97] B. I. Halperin and T. M. Rice, Solid State Phys. **21**, 115 (1968).
[98] Yu. V. Kopaev, Tr. Fiz. Inst. Akad. Nauk SSSR **86**, 3 (1975).
[99] G. Grüner, *Density Waves in Solids* (Addison-Wesley, Reading, MA, 1994).
[100] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1987).
[101] R. R. Guseinov and L. V. Keldysh, Zh. Eksp. Teor. Fiz. **63**, 2255 (1972).
[102] I. V. Krive, A. S. Rozhavskii, and I. O. Kulik, Fiz. Nizk. Temp. **12**, 1123 (1986).
[103] A. M. Gabovich and A. I. Voitenko, Phys. Rev. B **55**, 1081 (1997).
[104] *Problem of High-Temperature Superconductivity*, edited by V. L. Ginzburg and D. A. Kirzhnitz (Nauka, Moscow, 1977) (in Russian).
[105] K. Levin, D. L. Mills, and S. L. Cunningham, Phys. Rev. B **10**, 3821 (1974).
[106] A. M. Gabovich and E. A. Pashitskii, Fiz. Tverd. Tela (Leningrad) **17**, 1584 (1975).
[107] A. M. Gabovich and A. S. Shpigel, J. Low Temp. Phys. **51**, 581 (1983).
[108] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, NJ, 1963).
[109] J. E. Dowman, M. L. A. MacVicar, and J. R. Waldram, Phys. Rev. **186**, 452 (1969).
[110] M. Ledvij and R. A. Klemm, Phys. Rev. B **51**, 3269 (1995).
[111] Z. Yusof, J. F. Zasadzinski, L. Coffey, and N. Miyakawa, Phys. Rev. B **58**, 514 (1998).
[112] R. A. Klemm, Phys. Rev. B **67**, 174509 (2003).
[113] D. H. Douglass Jr., and L. M. Falicov, Prog. Low Temp. Phys. **4**, 97 (1964).
[114] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **51**, 1535 (1966).
[115] A. S. Alexandrov and A. F. Andreev, Europhys. Lett. **54**, 373 (2001).
[116] A. S. Alexandrov, J. Supercond. **18**, 603 (2005).
[117] J. E. Hoffman, K. McElroy, D-H. Lee, K. M. Lang, H. Eisaki, S. Uchida, and J. C. Davis, Science **297**, 1148 (2002).
[118] K. McElroy, R. W. Simmonds, J. E. Hoffman, D-H. Lee, J. Orenstein, H. Eisaki, S. Uchida, and J. C. Davis, Nature (London) **422**, 592 (2003).
[119] C. Howald, H. Eisaki, N. Kaneko, M. Greven, and A. Kapitulnik, Phys. Rev. B **67**, 014533 (2003).
[120] A. M. Gabovich and A. I. Voitenko, Phys. Rev. B **52**, 7437 (1995).
[121] Yu. I. Latyshev, P. Monceau, A. P. Orlov, S. Brazovskii, and T. Fournier, Supercond. Sci. Technol. **20**, S87 (2007).
[122] T. Machida, Y. Kamijo, K. Harada, T. Noguchi, R. Saito, T. Kato, and H. Sakata, J. Phys. Soc. Jpn. **75**, 083708 (2006).
[123] J. A. Robertson, S. A. Kivelson, E. Fradkin, A. C. Fang, and A. Kapitulnik, Phys. Rev. B **74**, 134507 (2006).
[124] D. Valdez-Balderas and D. Stroud, Phys. Rev. B **74**, 174506 (2006).
[125] A. Chang, Z. Y. Rong, Yu. M. Ivanchenko, F. Lu, and E. L. Wolf, Phys. Rev. B **46**, 5692 (1992).
[126] T. Cren, D. Roditchev, W. Sacks, J. Klein, J.-B. Moussy, C. Deville-Cavellin, and M. Laguës, Phys. Rev. Lett. **84**, 147 (2000).
[127] S. H. Pan, J. P. O'Neal, R. L. Badzey, C. Chamon, H. Ding, J. R. Engelbrecht, Z. Wang, H. Eisaki, S. Uchida, A. K. Gupta, K.-W. Ng, E. W. Hudson, K. M. Lang, and J. C. Davis, Nature (London) **413**, 282 (2001).
[128] K. M. Lang, V. Madhavan, J. E. Hoffman, E. W. Hudson, H. Eisaki, S. Uchida, and J. C. Davis, Nature (London) **415**, 412 (2002).
[129] K. McElroy, J. Lee, J. A. Slezak, D.-H. Lee, H. Eisaki, S. Uchida, and J. C. Davis, Science **309**, 1048 (2005).
[130] Y. Yamada and M. Suzuki, Physica C **426–431**, 364 (2005).
[131] D. Mihailovic and V. V. Kabanov, in *Superconductivity in Complex Systems, Structure and Bonding*, Vol. 114, edited by K. A. Muller and A. Bussmann-Holder (Springer-Verlag, Berlin, 2005), p. 331.
[132] H. Mashima, N. Fukuo, Y. Matsumoto, G. Kinoda, T. Kondo, H. Ikuta, T. Hitosugi, and T. Hasegawa, Phys. Rev. B **73**, 060502(R) (2006).
[133] A. C. Fang, L. Capriotti, D. J. Scalapino, S. A. Kivelson, N. Kaneko, M. Greven, and A. Kapitulnik, Phys. Rev. Lett. **96**, 017007 (2006).
[134] J.-X. Liu, J.-C. Wan, A. M. Goldman, Y. C. Chang, and P. Z. Jiang, Phys. Rev. Lett. **67**, 2195 (1991).
[135] Y. Yamada, K. Anagawa, T. Shibauchi, T. Fujii, T. Watanabe, A. Matsuda, and M. Suzuki, Phys. Rev. B **68**, 054533 (2003).
[136] G. Kinoda, S. Nakao, T. Motohashi, Y. Nakayama, K. Shimizu, J. Shimoyama, K. Kishio, T. Hanaguri, K. Kitazawa, and T. Hasegawa, Physica C **388–389**, 273 (2003).
[137] A. Sugimoto, S. Kashiwaya, H. Eisaki, H. Kashiwaya, H. Tsuchiura, Y. Tanaka, K. Fujita, and S. Uchida, Phys. Rev. B **74**, 094503 (2006).
[138] S. Kashiwaya, A. Sugimoto, H. Eisaki, T. Matsumoto, H. Kashiwaya, and Y. Tanaka, Physica C **445–448**, 146 (2006).
[139] N. A. Belous, A. E. Chernyakhovskii, A. M. Gabovich, D. P. Moiseev, and V. M. Postnikov, J. Phys. C **21**, L153 (1988).
[140] A. M. Gabovich and D. P. Moiseev, Usp. Fiz. Nauk **150**, 599 (1986).
[141] K. Anagawa, Y. Yamada, T. Shibauchi, M. Suzuki, and T. Watanabe, Appl. Phys. Lett. **83**, 2381 (2003).

[142] V. N. Zavaritsky, Phys. Rev. Lett. **92**, 259701 (2004).
[143] V. N. Zavaritsky, Phys. Rev. B **72**, 094503 (2005).
[144] A. Yurgens, D. Winkler, T. Claeson, S. Ono, and Y. Ando, Phys. Rev. Lett. **92**, 259702 (2004).
[145] V. M. Krasnov, M. Sandberg, and I. Zogaj, Phys. Rev. Lett. **94**, 077003 (2005).
[146] V. A. Drozd, S. O. Solopan, S. A. Nedil'ko, A. M. Gabovich, M. Pękała, and O. G. Dzyaz'ko, Ukr. Chem. J. **71**, 77 (2005).
[147] J. Bobroff, H. Alloul, S. Ouazi, P. Mendels, A. Mahajan, N. Blanchard, G. Collin, V. Guillen, and J.-F. Marucco, Phys. Rev. Lett. **89**, 157002 (2002).
[148] A. I. Golovashkin, L. N. Zherikhina, G. V. Kuleshova, A. M. Tskhovrebov, and M. L. Norton, Zh. Eksp. Teor. Fiz. **129**, 684 (2006).
[149] R. Collongues, *La Non-Stoechiometrie* (Masson, Paris, 1971).
[150] J. Zhang, Ismail, R. G. Moore, S.-C. Wang, H. Ding, R. Jin, D. Mandrus, and E. W. Plummer, Phys. Rev. Lett. **96**, 066401 (2006).
[151] T. S. Nunner, B. M. Andersen, A. Melikyan, and P. J. Hirschfeld, Phys. Rev. Lett. **95**, 177003 (2005).
[152] P. Richard, Z.-H. Pan, M. Neupane, A. V. Fedorov, T. Valla, P. D. Johnson, G. D. Gu, W. Ku, Z. Wang, and H. Ding, Phys. Rev. B **74**, 094512 (2006).
[153] A. V. Balatsky and J.-X. Zhu, Phys. Rev. B **74**, 094517 (2006).
[154] D. J. van Harlingen, Rev. Mod. Phys. **67**, 515 (1995).
[155] D. J. Scalapino, Phys. Rep. **250**, 329 (1995).
[156] M. L. Kulić, Phys. Rep. **338**, 1 (2000).
[157] W. Brenig, Phys. Rep. **251**, 153 (1995).
[158] N. Bulut, Adv. Phys. **51**, 1587 (2002).
[159] F. Mancini and A. Avella, Adv. Phys. **53**, 537 (2004).
[160] A.-M. S. Tremblay, B. Kyung, and D. Sénéchal, Fiz. Nizk. Temp. **32**, 561 (2006).
[161] P. Bourges, B. Keimer, S. Pailhès, L. P. Regnault, Y. Sidis, and C. Ulrich, Physica C **424**, 45 (2005).
[162] Ph. Bourges, B. Keimer, L. P. Regnault, and Y. Sidis, J. Supercond. **13**, 735 (2000).
[163] P. Dai, H. A. Mook, S. M. Hayden, G. Aeppli, T. G. Perring, R. D. Hunt, and F. Doğan, Science **284**, 1344 (1999).
[164] H. A. Mook, P. Dai, S. M. Hayden, G. Aeppli, T. G. Perring, and F. Doğan, Nature (London) **395**, 580 (1998).
[165] T. Timusk and B. Statt, Rep. Prog. Phys. **62**, 61 (1999).
[166] J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, Nature (London) **429**, 534 (2004).
[167] Yu. A. Izyumov, Usp. Fiz. Nauk **169**, 225 (1999).
[168] P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006).
[169] E. G. Maksimov, Usp. Fiz. Nauk **170**, 1033 (2000).
[170] E. G. Maksimov, O. V. Dolgov, and M. L. Kulić, Phys. Rev. B **72**, 212505 (2005).
[171] H.-Y. Kee, S. A. Kivelson, and G. Aeppli, Phys. Rev. Lett. **88**, 257002 (2002).
[172] A. S. Alexandrov and N. F. Mott, Rep. Prog. Phys. **57**, 1197 (1994).
[173] A. S. Alexandrov, Philos. Mag. B **81**, 1397 (2001).
[174] A. S. Alexandrov and C. Sricheewin, Europhys. Lett. **58**, 576 (2002).
[175] S. Pilgram, T. M. Rice, and M. Sigrist, Phys. Rev. Lett. **97**, 117003 (2006).
[176] D. J. Scalapino, Nat. Phys. **2**, 593 (2006).
[177] I. Martin and A. Balatsky, Physica C **357–360**, 46 (2001).
[178] B. M. Andersen, A. Melikyan, T. S. Nunner, and P. J. Hirschfeld, Phys. Rev. B **74**, 060501(R) (2006).
[179] A. M. Gabovich, E. A. Pashitskii, and A. S. Shpigel, Fiz. Tverd. Tela (Leningrad) **18**, 3279 (1976).
[180] J. F. Annett, Adv. Phys. **39**, 83 (1990).
[181] J. F. Annett, Contemp. Phys. **36**, 423 (1995).
[182] V. P. Mineev and K. V. Samokhin, *Intoduction to Unconventional Superconductivity* (Gordon and Breach, Amsterdam, 1999).
[183] N. P. Ong and P. Monceau, Phys. Rev. B **16**, 3443 (1977).
[184] Y. Nishihara, Y. Yamaguchi, T. Kohara, and M. Tokumoto, Phys. Rev. B **31**, 5775 (1985).
[185] A. Fang, C. Howald, N. Kaneko, M. Greven, and A. Kapitulnik, Phys. Rev. B **70**, 214514 (2004).
[186] C. Howald, P. Fournier, and A. Kapitulnik, Phys. Rev. B **64**, 100504(R) (2001).
[187] J.-X. Zhu, A. V. Balatsky, T. P. Devereaux, Q. Si, J. Lee, K. McElroy, and J. C. Davis, Phys. Rev. B **73**, 014511 (2006).
[188] J. F. Zasadzinski, L. Ozyuzer, N. Miyakawa, K. E. Gray, D. G. Hinks, and C. Kendziora, J. Phys. Chem. Solids **63**, 2247 (2002).
[189] A. D. Beyer, C.-T. Chen, and N.-C. Yeh, cond-mat/0610855 (unpublished).
[190] G. Deutscher, Fiz. Nizk. Temp. **32**, 740 (2006).
[191] T. Valla, A. V. Fedorov, J. Lee, J. C. Davis, and G. D. Gu, Science **314**, 1914 (2006).