Suppression of scintillations and beam wandering in free space gigabit rate optical communication based on spectral encoding of a partially coherent beam

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ABSTRACT

A new concept of a free-space, high-speed optical communication (FSOC) system based on spectral encoding of radiation from a broadband pulsed laser is developed. It is known that the intensity fluctuations of a partially coherent beam in combination with a time-averaging photodetector leads to a significant scintillation reduction with the corresponding improvement of the bit error rate by several orders of magnitude. Unfortunately, the time-averaging method cannot be applied directly to gigabit data rate communication. The main limitation of this method is related to the requirement that the correlation time between different spatially coherent spots be shorter than the response time of the photodetector**.** We propose to extend the technique of scintillation suppression, based on time averaging of a partially coherent beam, to gigabit data rate FSOC. In our approach, information is encoded in the form of amplitude modulation of the spectral components of the laser pulse which has a broad spectrum. To examine the intensity fluctuations of a partially coherent beam under the conditions of strong turbulence, we developed an asymptotic method for solution of the kinetic equation for the photon distribution function. We show that, for long distances, scintillations and beam wandering can be significantly suppressed.

Keywords: optical communication, atmospheric turbulence, scintillations, encoding.

1. INTRODUCTION

Free-space optical communication (FSOC) has data rate limitations due to atmospheric turbulence. Laser beams experience three major effects under the influence of turbulence. *First*, the beam phase front is distorted by fluctuations in the refractive index, causing intensity fluctuations or scintillations. *Second*, eddies whose size is greater than the beam diameter randomly deflect the laser beam as a whole; this phenomenon is called beam wandering. *Third*, propagation through turbulent atmosphere causes the laser beam to spread more than predicted by diffraction theory. Scintillations are the most severe problem and result in a significant increase of the bit error rate (BER) and consequent degradation of the laser communication system performance. For example, a gigabit data rate communication channel cannot operate with BER of 10^{-9} over distances more than 2.5 km, even for clear weather[1,2]. Several approaches have been developed to mitigate the effects of turbulence on laser communication. They include: aperture averaging, partially coherent beams, adaptive optics, and array receivers.

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(See the detailed reviews in[1,3].) Nevertheless, scintillations continue to limit the performance of FSOC, and new approaches are needed to overcome this limitation. It is well known that partially coherent beams (beams with multiple coherent spots in their transverse section) are less affected during propagation through atmospheric turbulence than a fully coherent beam[4-13]. Specifically, the additional beam spreading due to the atmospheric turbulence[4,5], the beam quality degradation [6], and the scintillation index[7-10] are less pronounced for a partially coherent beam compared with fully coherent beam. Recently the techniques of scintillation reduction based on the utilization of partially coherent beams were demonstrated[10-13]. To form partial coherence, authors of[10] used a static phase diffuser. Combining partially coherent beams with time-averaging leads to a significant scintillation reduction with the corresponding improvement of the BER by several orders of magnitude[10]. Another possibility is related to utilization of a spatial light modulator (SLM). The main advantage of SLM compared with a rotating phase diffuser is that the random phase distribution at the transmitter plane could change at higher rates. As we show in Section 4, higher SLMs frame rate corresponds to higher data rate of the communication channel. Authors of[11,12] proposed an alternative approach using multiple beams with different wavelengths. This approach was experimentally demonstrated[13] using a multiemitter beam, constructed by spatially combining outputs of several single mode fiber-coupled diode lasers. It was shown theoretically[11,12] and experimentally[13] that the scintillation index can be substantially reduced if individual beams overlapped at the detector aperture and were properly separated at the transmitter plane. Unfortunately, the time-averaging method cannot be applied directly to gigabit rate communication. The main limitation of this method is related to the requirement that the correlation time between different spatially coherent spots be shorter than the response time of the photodetector. This means that the SLM must have an operating frequency ν higher than the bandwidth of the photodetector, corresponding to its inverse response time $\nu \gg T^{-1}$. Since the photodetector bandwidth must be higher than the data rate of the communication channel ν_{COM}, $T^{-1} \gg \nu_{COM}$, the highest data rate is limited by the highest frequency of SLM, $\nu \gg \nu_{COM}$. To date, the highest frequency SLMs based on multiple quantum wells (MQW) can only operate at frequencies up to tens of MHz[14].

In the present paper we propose to extend the technique of scintillation suppression, based on time averaging of a partially coherent beam (TAPCB), to gigabit rate FSOC. Our idea is to combine TAPCB with a spectral encoding technique. Originally, spectral encoding was applied to fiber optics communication for code-division-multiple-access[15]. In this method, information is encoded in the form of amplitude modulation of the spectral components of the laser pulse which has a broad spectrum. For long-distance communication, the broad-spectrum light source could be a Ti: sapphire laser. For short-distance communication it could be an LED as well. Each pulse or sequence of pulses (depending on the averaging response time of the photosensor) can contain kilobits of data. If the pulse repetition rate is about 1 MHz, then the transmitted data rate is gigabits per second. SLMs based on MQW technology with a frame rate of several MHz are now available [14].

2. SCINTILLATIONS REDUCTION DUE TO TIME AVERAGING OF A PARTIALLY COHERENT BEAM

It is well-established that for long distances the scintillation index of plane and spherical waves propagating through atmospheric turbulence asymptotically tends to unity[16]. For an initially partially coherent beam, the asymptotic behavior depends on the relation between the correlation time of the source and the response time of the photodetector. If the average correlation time of two different coherent spots in the beam's cross section is shorter than the response time of the photodetector, then the scintillation index asymptotically tends to zero[17-20]. If the correlation time of the coherent spots is longer than the response time of the photodetector, then the scintillation index asymptotically tends to unity[20,21]. As was shown in[20], these properties of a partially coherent beam can be easily explained if we assume that the scintillations at the photodetector follow Gaussian statistics. Indeed, if the coherence radius, r_c, of the initial beam is significantly smaller than the beam radius, r_0, the process of propagation of the laser beam can be considered as the independent propagation of a large number of coherent beams. Consequently, the intensity fluctuations of each coherent region caused by atmospheric turbulence are statistically independent. With increasing the propagation distance, the individual coherent spots overlap due to diffraction effects. According to the Central Limit Theorem, the intensity, which is the result of the contributions of a large number of independent regions, has a normal statistical distribution. The suppression of scintillations in the signal measurements is strictly due to the unique properties of the Gaussian statistics. The fluctuations in the signal generated by a photodetector with slow response time are proportional to the following integral over light intensity absorbed during the response time:

$$\langle i(t)i(t)\rangle - \langle i(t)\rangle^2 \sim \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t} dt_2 h(t-t_1)h(t-t_2)\left[\langle I(t_1)I(t_2)\rangle - \langle I(t_1)\rangle\langle I(t_2)\rangle\right]. \qquad (1)$$

Here i(t) is the photocurrent, I(t) is the light intensity, and $h(t-t_{1,2})$ is the time response function of the photodetector. Usually $h(t-t_{1,2})$ has rather complex structure and can be characterized by two or even more response times related to different physical processes such as the electron-hole recombination, carriers transmit time, intrinsic capacitance and resistance, etc. Below, under the term response time T we mean a shortest characteristic time of the photodetector. According to the extended Huygens-Fresnel principle[1,22], the optical field at the receiver plane can be expressed in terms of the integral optical field at an intermediate plane:

$$E(\vec{r},L,t) \sim \iint_{\Sigma} d^2 s E(\vec{s},z,t) \exp\left[\frac{ik|\vec{s}-\vec{r}|}{2(L-z)} + i\Psi(\vec{s},\vec{r},t)\right], \qquad (2)$$

where $\Psi(\vec{s},\vec{r})$ is the complex phase of the wave propagating through the turbulent medium from the point (s,z) to the point (r,L). As follows from expression (2), the values of the averaging in expression (1) are of fourth order in the field moment:

$$\langle E(\vec{s}_1,z,t_1)E^*(\vec{s}_2,z,t_1)E(\vec{s}_3,z,t_2)E^*(\vec{s}_4,z,t_2)\rangle. \qquad (3)$$

For Gaussian statistics, this fourth order moment can be expressed in terms of the second order moments:

$$\langle E(\vec{s}_1,z,t_1)E^*(\vec{s}_2,z,t_1)E(\vec{s}_3,z,t_2)E^*(\vec{s}_4,z,t_2)\rangle =$$
$$\langle E(\vec{s}_1,z,t_1)E^*(\vec{s}_2,z,t_1)\rangle\langle E(\vec{s}_3,z,t_2)E^*(\vec{s}_4,z,t_2)\rangle + \langle E(\vec{s}_1,z,t_1)E^*(\vec{s}_4,z,t_2)\rangle\langle E(\vec{s}_3,z,t_2)E^*(\vec{s}_2,z,t_1)\rangle. \qquad (4)$$

The typical difference between the times, t_1 and t_2, in (1) can be estimated as $|t_1 - t_2| \sim T$. If the response time of the photodetector, T, exceeds the average correlation time between two coherent spots τ_c, $T \gg \tau_c$, the second term on the right-hand side of the expression (4) is equal to zero. As a result, from Eq. (1) we obtain $\langle I(t_1)I(t_2)\rangle = \langle I\rangle^2$. This shows that the scintillation index $\sigma^2 = \left(\langle I^2\rangle - \langle I\rangle^2\right) / \langle I\rangle^2$ is equal to zero. In the opposite case, when the correlation time is much longer than the photodetector response time, $T \ll \tau_c$, the second term in the expression (4) is equal to the first term, and the scintillation index is equal to unity.

As the above considerations show, in order to exploit the unique properties of Gaussian statistics, the time response of the photodetector must be much longer than the inverse frame rate of the SLM. Another requirement is that the number of individual coherent spots in the initial beam must be sufficiently large. In other words, the coherence radius, r_c, must be much smaller than the beam radius, r_0.

3. FOURTH-ORDER CORRELATION FUNCTION APPROACH

Our analysis is based on the equation for the fourth-order correlation function derived by Tatarskii in the Markov approximation[23,24]. The equation for the correlation function

$$\Gamma_4(\zeta;\vec{\rho}_1,\vec{\rho}_1^{'},\vec{\rho}_2,\vec{\rho}_2^{'}) = \left\langle E(\zeta,\vec{\rho}_1)E^*(\zeta,\vec{\rho}_1^{'})E(\zeta,\vec{\rho}_2)E^*(\zeta,\vec{\rho}_2^{'})\right\rangle \tag{5}$$

has the form

$$\frac{\partial \Gamma_4}{\partial \zeta} = \frac{i}{2q}(\Delta_1 + \Delta_2 - \Delta_1^{'} - \Delta_2^{'})\Gamma_4 - F(\zeta;\vec{\rho}_1,\vec{\rho}_1^{'},\vec{\rho}_2,\vec{\rho}_2^{'})\Gamma_4, \tag{6}$$

where $\zeta = x/L, \vec{\rho}_{1,2} = \vec{r}_{1,2}/\rho_0, \vec{\rho}'_{1,2} = \vec{r}^{'}_{1,2}\big/\rho_0$ (x is the longitudinal coordinate), $\vec{r}_{1,2}, \vec{r}^{'}_{1,2}$ are the transversal coordinates, L is the propagation length, ρ_0 is the normalizing transverse scale, which is chosen below, $q = k\rho_0^2\big/L$, where k is the wave number, and

$$\begin{aligned} F(\zeta,\vec{r}_1,\vec{r}_2,\vec{\rho}) &= H(\zeta,\vec{r}_1+\vec{\rho}/2) + H(\zeta,\vec{r}_1-\vec{\rho}/2) + H(\zeta,\vec{r}_2+\vec{\rho}/2) + H(\zeta,\vec{r}_2-\vec{\rho}/2) \\ &- H(\zeta,\vec{r}_1+\vec{r}_2) - H(\zeta,\vec{r}_1-\vec{r}_2). \end{aligned} \tag{7}$$

In the expression (7) we introduced the new variables

$$\begin{aligned} &\vec{r}_1 = \frac{1}{2}(\vec{\rho}_1 - \vec{\rho}_2 + \vec{\rho}_1^{'} - \vec{\rho}_2^{'}); \vec{r}_2 = \frac{1}{2}(\vec{\rho}_1 - \vec{\rho}_2 - \vec{\rho}_1^{'} + \vec{\rho}_2^{'}), \\ &\vec{\rho} = \vec{\rho}_1 + \vec{\rho}_2 - \vec{\rho}_1^{'} - \vec{\rho}_2^{'};\ R = \frac{1}{4}(\vec{\rho}_1 + \vec{\rho}_2 + \vec{\rho}_1^{'} + \vec{\rho}_2^{'}). \end{aligned} \tag{8}$$

In these new variables, Eq. (6) takes the form

$$\frac{\partial \Gamma_4}{\partial \zeta} = \frac{i}{q}(\nabla_R\nabla_\rho + \nabla_{r_1}\nabla_{r_2})\Gamma_4 - F(\zeta,\vec{r}_1,\vec{r}_2,\vec{\rho})\Gamma_4, \tag{9}$$

where

$$H(\zeta,\vec{\rho}) = 8\iint \Phi_n(\zeta,\vec{\kappa})[1-\cos\vec{\kappa}(\vec{\rho}_1-\vec{\rho}_2)]d^2\vec{\kappa}\ , \tag{10}$$

and $\Phi_n(\zeta,\vec{\kappa})$ is the spectral density of the structure function of the refractive index, which is given by

$$\left\langle \delta n(\zeta,\vec{\rho})\delta n(\zeta,\vec{\rho}')\right\rangle = 2\pi\delta(\zeta-\zeta')\iint \Phi_n(\zeta,\vec{\kappa})\exp(-\vec{\kappa}(\vec{\rho}-\vec{\rho}'))d^2\kappa\,. \tag{11}$$

Following Tatarskii[24], we chose the spectral density of the structure function of the refractive index in the form

$$\Phi_n(\zeta,\vec{\kappa}) = 0.033 C_n^2 \kappa^{-11/3} \exp\left(-\frac{\kappa^2}{\kappa_m^2}\right). \tag{12}$$

In this case the analytic approximations for the function $H(\zeta,\vec{\rho})$ have the form

$$H(\zeta,\vec{\rho}) = \begin{cases} 1.64 C_n^2 k^2 \rho_0^2 \rho^2 l_0^{-1/3}, \text{ for } \rho \ll \dfrac{l_0}{\rho_0}, \\ 1.24 C_n^2 k^2 \rho_0^{5/3} \rho^{5/3}, \text{ for } \rho \gg \dfrac{l_0}{\rho_0}, \end{cases} \tag{13}$$

where $l_0 = 5.92\kappa_m$. The transverse scale, ρ_0, is the scale of variation of the phase structure function of the plane waves corresponding to the path L. It is defined by the equation[23] $1.64 C_n^2 k^2 L \rho_0^2 l_0^{-1/3} = 1$.

According to the procedure discussed in Section 2, we assume that the light source emits a partially coherent light with Gaussian statistics at the source plane, $\zeta = 0$. Hence, the fourth-order correlation function can be expressed in terms of the second order correlation functions

$$\begin{aligned} \Gamma_{4,0}(\vec{\rho}_1,\vec{\rho}_1^{'},\vec{\rho}_2,\vec{\rho}_2^{'}) = \left\langle E(\zeta=0,\vec{\rho}_1)E^*(\zeta=0,\vec{\rho}_1^{'})\right\rangle \left\langle E(\zeta=0,\vec{\rho}_2)E^*(\zeta=0,\vec{\rho}_2^{'})\right\rangle + \\ \left\langle E(\zeta=0,\vec{\rho}_1)E^*(\zeta=0,\vec{\rho}_2^{'})\right\rangle \left\langle E(\zeta=0,\vec{\rho}_2)E^*(\zeta=0,\vec{\rho}_1^{'})\right\rangle, \end{aligned} \tag{14}$$

where the normalized second order correlation function is given by the expression

$$\Gamma_{2,0}(\vec{\rho}_{1,2},\vec{\rho}'_{1,2}) = \left\langle E(\zeta=0,\vec{\rho}_{1,2})E^*(\zeta=0,\vec{\rho}'_{1,2})\right\rangle = \exp\left(-\frac{\vec{\rho}_{1,2}^2+\vec{\rho}_{1,2}'^2}{2r_0^2}\right)\exp\left(-\frac{(\vec{\rho}_{1,2}-\vec{\rho}'_{1,2})^2}{r_c^2}\right). \tag{15}$$

In Eq. (15), r_0 is the beam radius and r_c is the coherence radius. We consider the case of small coherence radii in comparison with the beam radius: $r_c \ll r_0$. The conventional approach to the problem of laser beam propagation is based on the assumption of small deviations of the beam parameters from those which correspond to free-space propagation[1]. This approach is limited to the conditions of weak turbulence or short propagation lengths. Another approach was developed by Yakushkin[17]. His approach is based on the fact that for any relatively long distance, the coherence radius is smaller than the beam radius. Starting with the exact solution of Eq. (9) for a beam with $r_c = 0$, Yakushkin developed a perturbation theory in which the small parameter, $r_c/r_0 \ll 1$, is the ratio of the coherence radius r_c to the beam radius r_0. In[17] the case of an initially fully coherent beam was considered. Thus, his theory was actually an asymptotic theory, applicable to relatively long distances. In our case, we have initially a partially coherent beam. Propagating through the turbulent atmosphere, the partially coherent beam experiences two opposite effects. The first effect is an increase of the coherence radius due to diffraction. The contribution of this effect is of the order $\delta r_c \sim \lambda / r_c L$. The second effect is a decrease of the coherence spot due to the influence of the turbulence. This effect is characterized by the scale of the phase structure function, $\rho_0 = (1.64 C_0^2 k^2 l_0^{-1/3} L)^{1/2}$. If the turbulence is strong enough, the decrease of the coherence radius dominates even at relatively small propagation lengths. For example, for the values of parameters $C_n^2 = 10^{-13}\ \text{m}^{-2/3}$, $r_c = 10^{-2}$ m, $l_0 = 10^{-3}$ m, $\kappa = 2\pi/\lambda = 4\times10^6\ \text{m}^{-1}$, the propagation length where $\delta r_c \sim \rho_0 \approx 2\times10^{-2}$ m, is $L = 115.6$ m. For the beam radius, $r_0 = 10^{-1}$ m, the assumption $r_c \ll r_0$ is correct at this distance.

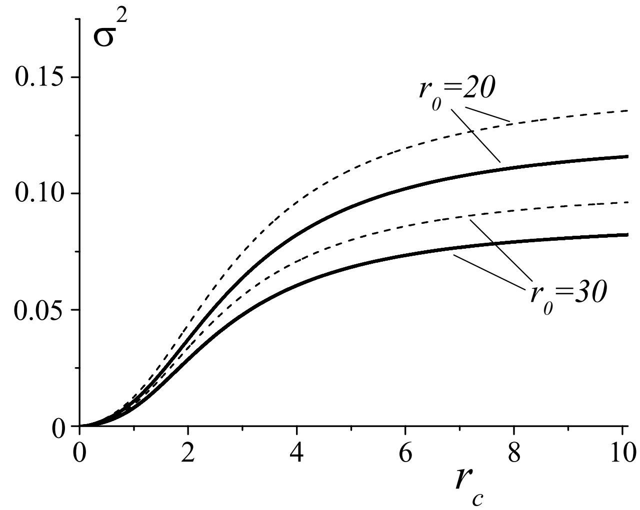

Figure 1. The dependence of the scintillation index on the initial coherence radius (for the center of the beam). In the dimensionless quantities $r_{c,0}$ the unit is $\rho_0 = [1.64 C_n^2 k^2 L l_0^{-1/3}]^{-1/2}$. The following values of the parameters were used: $L = 3.7 km$, $C_n^2 = 10^{-13} m^{-2/3}$ (dashes lines), $L = 10 km$, $C_n{}^2 = 1.4\times10^{-14} m^{-2/3}$ (solid lines), $\lambda = 1.55 \mu m$, $l_0 = 2\times10^{-2} m$.

Hence, in the case of strong turbulence, our approach is applicable to any distance. The formula (16) describes the case of finite coherence radius, when the scintillation index is different from zero even in the case of very strong turbulence.

$$\sigma^2 = 0.68\alpha_0(\xi=1) Q^{1/6} q^{1/6} \int_0^1 d\xi \frac{\left[\dfrac{0.286}{\beta(\xi)} - \dfrac{0.43}{\gamma(\xi)} + \dfrac{0.157\mu^2(\xi)}{\gamma^2(\xi)\alpha_2(\xi)}\right]}{(1-\xi^2)\alpha_0(\xi)\gamma(\xi)\beta(\xi)\alpha_2^{5/6}(\xi)}, \tag{16}$$

where

$$Q = \frac{kl_0^2}{L};\ \alpha_0 = \xi + \frac{1}{r_f^2} + \frac{q^2 r_0^2}{4\xi^2};\quad \frac{1}{r_f^2} = \frac{1}{r_c^2} + \frac{1}{4r_0^2};\ \mu = \frac{1}{1-\xi} + \frac{1}{2\xi\alpha_0 r_f^2};\quad \nu = \frac{2}{\alpha_0 r_f^2}\left(\xi + \frac{q^2 r_0^2}{4\xi^2}\right);$$

$$\beta = \nu + 1 - \xi + \frac{\alpha_0 \xi^2 \mu^2}{2};\quad \gamma = \nu + 2(1-\xi);\quad \alpha_2 = \frac{\mu^2}{4\gamma} + \frac{1}{8\xi^2\alpha_0}.$$

The scintillation index decreases as the initial coherence radius, r_c, decreases, as can be seen in Fig. 1. For a coherence radii less than 4, the scintillation index has a quadratic-like dependence. For a certain value of the coherence radius, a larger beam radius corresponds to a smaller scintillation index. Actually, the scintillation index decreases linearly with an inverse number of coherent spots: $\sigma^2 \sim N_c^{-1} = r_c^2 / r_0^2$. Thus the scintillation index decreases by an order of magnitude as the coherence radius r_c decreases from 3 to 1 (see Fig. 1).

4. KINETIC EQUATION FOR THE PHOTON DISTRIBUTION FUNCTION

In this section we obtain the scintillation index for the case of a PCB using an alternative theoretical approach. The beam characteristics at an arbitrary instant, t, can be described in terms of the distribution function of photons
The Hamiltonian for photons in a medium with a fluctuating refractive index is given by

$$H = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} b_{\mathbf{k}}^{+} b_{\mathbf{k}} - \sum_{\mathbf{k},\mathbf{k}'} \hbar\omega_{\mathbf{k}} n_{\mathbf{k}'} b_{\mathbf{k}}^{+} b_{\mathbf{k}+\mathbf{k}'}, \tag{17}$$

where the two terms on the right-hand side describe photons in a vacuum and the effect of refractive index fluctuations, respectively; $b_{\mathbf{k}}^{+}$ and $b_{\mathbf{k}}$ are creation and annihilation operators of photons with momentum $\mathbf{k}$, $\hbar\omega_{\mathbf{k}} \equiv \hbar ck$ is the photon energy, c is the speed of light in a vacuum, and $n_{\mathbf{k}}$ is the Fourier transform of the refractive index fluctuations $\delta n(\mathbf{r})$. The Fourier transform is defined by

$$n_{\mathbf{k}} = \frac{1}{V} \int dV e^{i\mathbf{kr}} \delta n(\mathbf{r}), \tag{18}$$

where $V \equiv L_x L_y L_z$ is the normalizing volume.
Eq. (17) follows from the representation of the energy of the random medium plus electromagnetic field given in[25] (Chapter 15) in the limit of small wave-vectors $\mathbf{k}'$ ($k' \ll k$) and an atmosphere refractive index close to unity ($n(\mathbf{r}) - 1 << 1$). The former means that the scale of the spatial inhomogeneity of turbulence is much greater than the wavelength of the radiation. For simplicity, we consider here only the case of polarized light with a fixed polarization throughout the distance of propagation. Depolarization effects due to atmospheric turbulence are very small. Also, the terms describing the zero-point electromagnetic energy are omitted in Eq. (17).
The photon distribution function is defined by analogy with the distribution functions for electrons, phonons, etc.:

$$f(\mathbf{r},\mathbf{q},t) = \frac{1}{V} \sum_{\mathbf{k}} e^{-i\mathbf{kr}} b_{\mathbf{q}+\mathbf{k}/2}^{+} b_{\mathbf{q}-\mathbf{k}/2}. \tag{19}$$

The distribution function will be used to describe beams with characteristic sizes much larger than the photon wave-length. Thus, we will restrict a sum over $\mathbf{k}$ by some $\mathbf{k}_0$ ($k < k_0 << q_0$, where q_0 is the wave vector corresponding to the central frequency ω_0 of the radiation, $\omega_0 = ck_0$). At the same time, k_0 is chosen to be large enough to sample the spatial variation of the light intensity. After integration of the operator function $f(\mathbf{r},\mathbf{q},t)$ over the volume V, we obtain the operator for the total number of photons with momentum $\mathbf{q}$:

$$\int dV f(\mathbf{r},\mathbf{q},t) = b_{\mathbf{q}}^{+} b_{\mathbf{q}}. \tag{20}$$

Similarly, the quantity obtained after a summation over $\mathbf{q}$ may serve as a photon number operator representing the number of photons in the small volume, $(2\pi/k_0)^3$. This is very similar to the Mandel operator[26] (Chapter 12) introduced in[27].

Here and in the remainder of this paper, we use the Heisenberg representation for all operators. Thus, the evolution of $f(\mathbf{r},\mathbf{q},t)$ is determined by the commutator with the total Hamiltonian:

$$\partial_t f(\mathbf{r},\mathbf{q},t)=\frac{1}{i\hbar}[f(\mathbf{r},\mathbf{q},t),H]. \tag{21}$$

Eq. (21) can be written explicitly as

$$\partial_t f(\mathbf{r},\mathbf{q},t)+\mathbf{c}_{\mathbf{q}}\partial_{\mathbf{r}} f(\mathbf{r},\mathbf{q},t)-i\omega_0\sum_{\mathbf{k}} e^{-i\mathbf{kr}} n_{\mathbf{k}}\left[f(\mathbf{r},\mathbf{q}+\frac{\mathbf{k}}{2},t)-f(\mathbf{r},\mathbf{q}-\frac{\mathbf{k}}{2},t)\right]=0. \tag{22}$$

Considering the characteristic values of the photon momentum to be much greater than the wave vectors of turbulence, we can express the difference of functions in square brackets by the corresponding derivative. A detailed discussion of this approximation is given in the text below. Then, after summing over $\mathbf{k}$, we obtain

$$\{\partial_t+\mathbf{c}_{\mathbf{q}}\partial_{\mathbf{r}}+\mathbf{F}(\mathbf{r})\partial_{\mathbf{q}}\}f(\mathbf{r},\mathbf{q},t)=0, \tag{23}$$

where $\mathbf{c}_{\mathbf{q}}=\partial_{\mathbf{q}}\omega_{\mathbf{q}}$ and $\mathbf{F}(\mathbf{r})=\omega_0\partial_{\mathbf{r}}n(\mathbf{r})$. As we see, the photon distribution function is governed by a kinetic equation in which the random force $\mathbf{F}(\mathbf{r})$ originates from atmospheric turbulence.

The distribution function determines the density of photons at a point ($\mathbf{r}$, $\mathbf{q}$) of the phase space at time t. Eq. (23) may be interpreted as the equation governing the evolution of a particle distribution function in which the state of each particle is described by the individual coordinate $\mathbf{r}$ and the momentum $\mathbf{q}$. The trajectories of these particles may be obtained from the solution of the equations of motion:

$$\frac{\partial\mathbf{r}(t)}{\partial t}=\mathbf{c}_{\mathbf{q}(t)},$$

$$\frac{\partial\mathbf{q}(t)}{\partial t}=\mathbf{F}(\mathbf{r}(t)). \tag{24}$$

Then, the general solution of Eq. (23) is given by

$$f(\mathbf{r},\mathbf{q},t)=\phi\left\{\mathbf{r}-\int_0^t dt'\,\frac{\partial\mathbf{r}(t')}{\partial t'};\mathbf{q}-\int_0^t dt'\,\frac{\partial\mathbf{q}(t')}{\partial t'}\right\}, \tag{25}$$

where the function $\phi(\mathbf{r},\mathbf{q})$ is the "initial" value of $f(\mathbf{r},\mathbf{q},t)$, i.e.

$$\phi(\mathbf{r},\mathbf{q})=\frac{1}{V}\sum_{\mathbf{k}} e^{-i\mathbf{kr}}(b^{+}_{\mathbf{q}+\mathbf{k}/2}b_{\mathbf{q}-\mathbf{k}/2})|_{t=0}\equiv\sum_{\mathbf{k}} e^{-i\mathbf{kr}}\phi(\mathbf{k},\mathbf{q}), \tag{26}$$

and the "trajectories" $\mathbf{r}(t')$ and $\mathbf{q}(t')$ pass through the point $\mathbf{r},\mathbf{q}$ at $t'=t$ [i.e. $\mathbf{r}(t'=t)=\mathbf{r},\mathbf{q}(t'=t)=\mathbf{q})$]. As one can see, the photon distribution function at an arbitrary instant t is expressed via the operators $b^{+}_{\mathbf{q}},\ b_{\mathbf{q}}$ defined for some fixed $t_0=0$. It is convenient to set $t-t_0=z/c$. Thus, t_0 is the instant when photons exit from the source. The initial photon statistics (at t_0), determined by the source properties, is assumed to be given.

We consider here the propagation of light beams with narrow spread (paraxial beams). In this case, $k_\perp, q_\perp \ll q_0$, where index ($_\perp$) means perpendicular to the direction of propagation (the z-axis). The relative effects of turbulence on q_z are negligible because of the large value of q_0. At the same time, $q_\perp$, which determines beam divergence, can be increased

considerably due to turbulence (compared to the initial value). Therefore, beam characteristics should be modified significantly for the case of long-distance propagation.
It follows from Eq. (24) that the evolution of transverse photon momentum is given by

$$\mathbf{q}_\perp(t') = \mathbf{q}_\perp + \int_t^{t'} dt'' \mathbf{F}_\perp[\mathbf{r}(t'')]. \tag{27}$$

Similarly, we obtain an expression for $\mathbf{r}(t')$:

$$\mathbf{r}(t') = \mathbf{r} - \mathbf{c}_\mathbf{q}(t-t') - \frac{c}{q_0}\int_t^{t'} dt''(t''-t')\mathbf{F}_\perp[\mathbf{r}(t'')]. \tag{28}$$

Then, Eq. (25) can be written as

$$f(\mathbf{r},\mathbf{q},t) = \phi\{\mathbf{r} - \mathbf{c}_\mathbf{q}t + \frac{c}{q_0}\int_0^t dt' t' \mathbf{F}_\perp[\mathbf{r}(t')]; \mathbf{q} - \int_0^t dt' \mathbf{F}_\perp[\mathbf{r}(t')]\}. \tag{29}$$

A regular iterative procedure is applicable here to expand $\mathbf{r}(t')$ in powers of $\mathbf{F}$. Then, substituting the explicit terms $\mathbf{r}(t')$ into Eq. (29), we will obtain the solution of the problem. In particular, the first and second moments of f, which describe beam spreading and intensity fluctuations, can be calculated. Applying this perturbation method, we can investigate the effects of the initial partial (spatial) coherence on beam spreading and scintillation.

The intensity of radiation in the z-direction at $\mathbf{r}$ can be presented in the form

$$I(\mathbf{r}) = c\sum_\mathbf{q} \hbar\omega_\mathbf{q} f(\mathbf{r},\mathbf{q},t). \tag{30}$$

This can be rewritten as

$$I(\mathbf{r}) = c\hbar\omega_0 \sum_{\mathbf{q},\mathbf{k}} e^{-i\mathbf{k}\{\{\mathbf{r}-\mathbf{c}(\mathbf{q})t+(c/q_0)\int_0^t dt' t' \mathbf{F}_\perp[\mathbf{r}(t')]\}} \phi_\mathbf{k}\{\mathbf{q} - \int_0^t dt' t' \mathbf{F}_\perp[\mathbf{r}(t')]\}. \tag{31}$$

Let us restrict ourselves to terms linear in $\mathbf{F}$ terms in Eq. (28). In other words, we set $\mathbf{r} - \mathbf{c}_\mathbf{q}(t-t')$ for $\mathbf{r}(t')$ in arguments of $\mathbf{F}_\perp$ in Eq. (31). After changing variables, $\mathbf{q}_\perp - \int_0^t dt' t' \mathbf{F}_\perp[\mathbf{r}(t')] \to \mathbf{q}_\perp$, and using the relation $z = ct$, Eq. (31) is transformed to

$$I(\mathbf{r}) = c\hbar\omega_0 \sum_{\mathbf{q},\mathbf{k}} e^{-i\mathbf{k}_\perp\{\mathbf{r}_\perp - \mathbf{q}_\perp(z/q_0) + (c/q_0)\int_0^t dt'(t-t')\mathbf{F}_\perp[\mathbf{r}-c(\mathbf{q})(t-t')]\}} \phi_\mathbf{k}(\mathbf{q}). \tag{32}$$

The stochastic variables $\mathbf{F}$ and $\phi_\mathbf{k}(\mathbf{q})$, which are of different nature, are not correlated in Eq. (32). Averaging of each factor in the sum can be performed independently because the source fluctuations and the refractive index fluctuations are not correlated. Thus, we have

$$\langle I(\mathbf{r})\rangle = c\hbar\omega_0 \sum_{\mathbf{q},\mathbf{k}} \left\langle e^{-i\mathbf{k}_\perp\{\mathbf{r}_\perp - \mathbf{q}_\perp(z/q_0) + (c/q_0)\int_0^t dt' t' \mathbf{F}_\perp[\mathbf{r}-c(\mathbf{q})t']\}} \right\rangle \langle \phi_\mathbf{k}(\mathbf{q})\rangle. \tag{33}$$

In Eq. (33) and throughout this paper, we will calculate average values of functionals of δn. This can be carried out when the statistics of δn is known. Usually, δn is assumed to be a Gaussian random variable with known covariance

$\left\langle \delta n(\mathbf{r})\delta n(\mathbf{r}')\right\rangle$. The covariance is defined by its Fourier transform, $\psi(\mathbf{g})$, with respect to the difference $\mathbf{r}-\mathbf{r}'$. The dependence $\psi(\mathbf{g})$ is often approximated by the von Karman formula

$$\psi(\mathbf{g}) = 0.033 C_n^2 \frac{exp[-(g l_0/2\pi)^2]}{[g^2 + L_0^{-2}]^{11/6}}. \tag{34}$$

The quantities L_0 and l_0 are the outer and inner scale sizes of the turbulent eddies, respectively. In atmospheric turbulence, L_0 may range from 1 to 100 meters, and l_0 is usually on the order of several millimeters. C_n^2 is known as the index-of-refraction structure constant. In most physically important cases the quantity L_0^{-2} in the denominator of Eq. (34) can be omitted. In this case, the von Karman spectrum is reduced to the Tatarskii spectrum.
Using the explicit form for turbulence fluctuations (Eq. 34), the last equation becomes

$$\left\langle I(\mathbf{r})\right\rangle = c\hbar\omega_0 \sum_{\mathbf{q},\mathbf{k}} e^{-i\mathbf{k}_\perp[\mathbf{r}_\perp - \mathbf{q}_\perp(z/q_0)] - k_\perp^2 z^3 T} \left\langle \phi_{\mathbf{k}}(\mathbf{q})\right\rangle, \tag{35}$$

where the effect of turbulence is represented by a quantity $T = 0.558 C_n^2 l_o^{-1/3}$.

Mathematically, the effect of the phase diffuser may be taken into account by introducing the multiplier $e^{-i\varphi(\mathbf{r}_\perp)}$. Then, considering Φ to be a Gaussian-type function ($\Phi = \frac{\sqrt{2}}{\sqrt{\pi} r_0} e^{-r_\perp^2/r_0^2}$), we obtain

$$\left\langle \phi_{\mathbf{k}}(\mathbf{q})\right\rangle = \frac{2\pi r_1^2}{V L_x L_y} \left\langle b^+ b\right\rangle e^{-k^2 (r_0^2/8) - q^2 (r_1^2/2)}, \tag{36}$$

where $r_1^2 = r_0^2/(1 + 2 r_0^2 \lambda_c^{-2})$ and $< b^+ b >= |\beta|^2$ for the coherent state $|\beta>$ of the laser radiation. Here, symbols $\mathbf{q}$ and $\mathbf{k}$ are the perpendicular components of the wave vectors. As we see, the effect of partial coherence is represented by the value of r_1^2. Using Eqs. (35) and (36), it is found that

$$\left\langle I(\mathbf{r}=0)\right\rangle = I_0 \left[1 + \frac{4z^2}{q_0^2 r_0^2 r_1^2} + \frac{8 z^3 T}{r_0^2}\right]^{-1}, \tag{37}$$

where I_0 is equal to $\left\langle I(\mathbf{r}=0)\right\rangle$ at $\mathbf{r}_\perp = 0$ and $z = 0$. Eq. (**37**) coincides with the corresponding Eq. (39) of [4] when C_n^2 does not depend on the distance z and $r_1 = r_0$.

The intensity for arbitrary $\mathbf{r}_\perp$ can be obtained from Eq. (**37**) by multiplying its right-hand side by the factor $exp\left\{-\frac{2r_\perp^2}{r_0^2}\left[1 + \frac{4z^2}{q_0^2 r_0^2 r_1^2} + \frac{8z^3 T}{r_0^2}\right]^{-1}\right\}$. The beam radius R, defined as,

$$R^2 = \frac{\int d\mathbf{r}_\perp r_\perp^2 \left\langle I(r_\perp)\right\rangle}{\int d\mathbf{r}_\perp \left\langle I(r_\perp)\right\rangle}, \tag{38}$$

is given by

$$R^2 = \frac{r_0^2}{2}\left[1 + \frac{4z^2}{q_0^2 r_0^2 r_1^2} + \frac{8z^3 T}{r_0^2}\right]. \quad \mathbf{(39)}$$

The problems related to the suppression of wandering in the PCB were considered in[28]. In particular, the effect of a random phase screen on laser beam wander in a turbulent atmosphere was studied theoretically. The method presented above was used to describe the photon kinetics of both weak and strong turbulence. By bringing together analytical and numerical calculations, we have obtained the variance of beam centroid deflections caused by scattering off turbulent eddies. It is shown that an artificial distortion of the initial coherence of the radiation can be used to decrease the wandering effect. The physical mechanism responsible for this reduction and the applicability of our approach are discussed in[28].

The dependence of the beam radius on coherence radius r_1 is presented on Fig. 2. The intensity fluctuations are determined by the expression

$$\left\langle : I^2(\mathbf{r}) : \right\rangle = \frac{(c\hbar\omega_0)^2}{V^2} \sum_{\mathbf{q},\mathbf{k}} \sum_{\mathbf{q}',\mathbf{k}'} e^{-i(\mathbf{k}+\mathbf{k}')\mathbf{r}} \left\langle b^+_{\mathbf{q}+\mathbf{k}/2} b^+_{\mathbf{q}'+\mathbf{k}'/2} b_{\mathbf{q}'-\mathbf{k}'/2} b_{\mathbf{q}-\mathbf{k}/2} \right\rangle, \quad \mathbf{(40)}$$

where the symbol $\langle : \ : \rangle$ means the normal ordering of the creation and annihilation operators. (See[27] for more detail.)

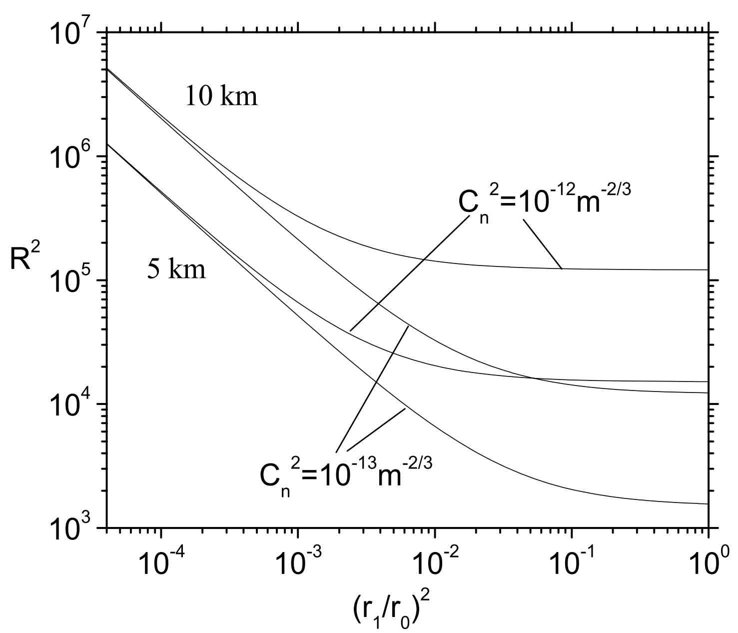

Figure 2. The dependence of beam radius on the coherence length parameter $(r_1/r_0)^2$ for $q_0 = 10^7 m^{-1}$, $(l_0 / 2\pi) = 10^{-3} m$.

Detailed calculations of the intensity fluctuations were made in[19]. In the case of weak turbulence, the expression for the scintillation index was derived analytically

$$\sigma^2 = \sigma_1^2 K(z, r_0, r_1), \quad \textbf{(41)}$$

where σ_1^2 is the Rytov variance defined by $\sigma_1^2 = 1.23 C_n^2 q_0^{7/6} z^{11/6}$ and

$$K(z,\rho_0,\rho_1) = 4.24\int_0^1 d\tau \int_0^\infty dx x^{-8/3} exp\left\{-x^2\left[\frac{q_0 l_0^2}{4\pi^2 z} + \tau^2\frac{\rho_0^2+\rho_1^2}{4+\rho_0^2\rho_1^2}\right]\frac{q_0 l_0^2}{4\pi^2 z} + \tau^2\frac{\rho_0^2+\rho_1^2}{4+\rho_0^2\rho_1^2}\right\} \quad \textbf{(42)}$$

$$\times sin^2\left(\frac{\tau x^2}{2} - \frac{2\tau^2 x^2}{4+\rho_0^2\rho_1^2}\right),$$

with $\rho_{0,1}^2 = r_{0,1}^2 q_0/z$.

The case of strong turbulence is more complex, and we present here only the results of numerical calculations.

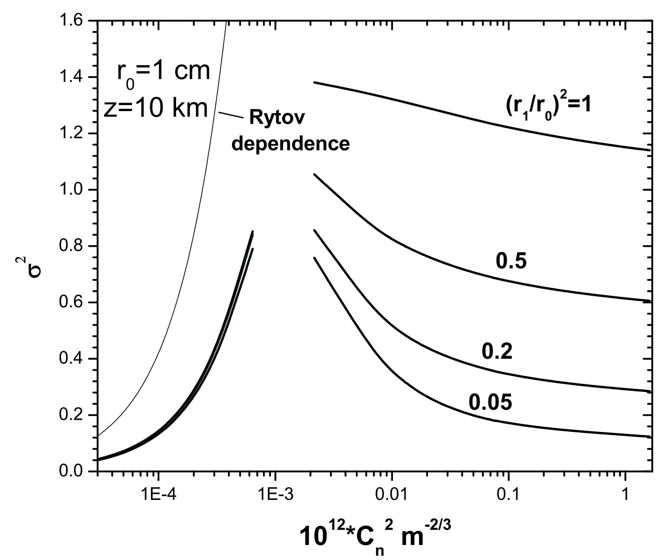

Figure 3. Plots $\sigma^2(C_n^2)$ for different initial coherence $(r_1/r_0)^2$; q_0 and l_0 are as in Fig. 2.

Fig. 3 shows the dependence of scintillation index on the turbulence. Two sets of curves show the solutions for weak and strong turbulences, respectively. A well-pronounced effect of decreasing σ^2 with a decrease of the initial coherence can be seen in the range of strong turbulence.

5. DESIGN OF AN OPTICAL SYSTEM BASED ON SPECTRAL AMPLITUDE ENCODING OF A BROAD BAND PULSED LASER

We propose to encode digital data in the spectrum of a wide-band source such as a Ti: sapphire laser. We assume that this laser operates at a high repetition rate. Usually Ti: sapphire lasers can operate at a repetition rate in a broad range from a few Hz up to GHz. If each series of N laser pulses (the number of pulses depends on the averaging time of the photosensor) contains kilobits of data and the series repetition rate is several MHz, then the data rate is a few Gbps. Usually, information is encoded as an amplitude modulation in time of a continuous wave laser beam. In this case, intensity fluctuations make significant contributions to the BER. Our spectral domain encoding technique is less sensitive to intensity and phase fluctuations, because the information is decoded in a massively parallel way using a relatively slow photosensor, which minimizes the scintillations by time averaging. Spectral-domain encoding can be achieved using a wide-band Ti: sapphire laser with an electro-optical SLM, as demonstrated in Fig. 4. The spectrum of each laser pulse is dispersed spatially and then encoded by the SLM, whose pixels can be turned on and off. The light traversing the SLM will have certain spectral bins turned on or off, depending on whether the corresponding pixels of the SLM are on or off. In our approach, the spatial coherence of the initial beam is also formed by the SLM. The encoded signal is sent through a second SLM which modifies the transverse coherence of the beam. The optimal value of the radius of coherence, r_c, is maintained by using a feedback loop between the SLM and the photosensor (not shown). At the receiver, the wavelength-modulated signal is dispersed in an optical spectrometer, *i.e.* a monochromator, and detected by a high-speed charge-coupled device (CCD). The electronic signal from the CCD is then processed by a high-speed data processing unit. Note, that in our method the spectral grating spreads the spectrum of the laser pulse along a single coordinate. Therefore we need a single array LSM for encoding and a single array CCD for sensing.

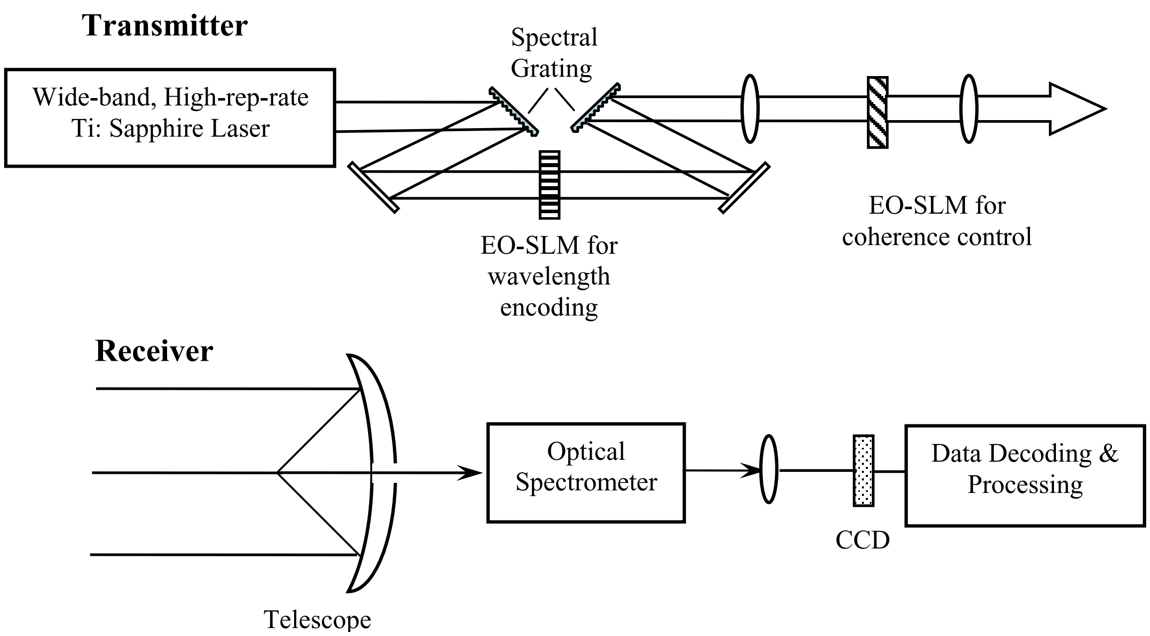

Fig. 4. Schematic for wide-band laser communication

Let us estimate the values of parameters needed to achieve Gbps rate. The grating dispersion is described by the grating equation

$$\sin\theta_{inc} - \sin\theta_{diff} = m\lambda/d\,, \qquad \mathbf{(43)}$$

where θ_{inc} is the incident angle, θ_{diff} is the diffraction angle, m is the diffraction order, and d is the grating period. Usually, the incident angle is fixed $\theta_{inc} = const$. Then the angle variation $\delta\theta_{diff}$ as a function of the wavelength variation $\delta\lambda$ is

$$\delta\theta_{diff} = m\delta\lambda / d\cos\theta_{diff} \quad \textbf{(44)}$$

The spatial image at the SLM plane of the spectral interval, corresponding to the distance between two neighboring bits, must exceed the size of individual pixel of the SLM, l_{pxl}. Then $\delta\theta_{diff} F \geq l_{pxl}$, where F is the focal distance of the imaging lens. From the expression **(44)** we obtain the following estimate of the spectral interval

$$\delta\lambda = l_{pxl} d\cos\theta_{diff} / Fm. \quad \textbf{(45)}$$

Using **(45)**, we estimate the information capacity, M, of the laser pulse which has the spectral width $\Delta\lambda$,

$$M = \Delta\lambda / \delta\lambda = \Delta\lambda Fm / l_{pxl} d\cos\theta_{diff}\ . \quad \textbf{(46)}$$

For the values of parameters $\Delta\lambda = 40nm$ (for the wavelength $\lambda = 1.55\mu m$ this spectral width corresponds to a pulse duration 200 fs), $F = 10cm$, $m = 2$, $l_{pxl} = 10\mu m$, $d^{-1} = 1.5\times10^3 mm^{-1}$, $\cos\theta_{diff} = 0.6$, we obtain $M = 2\times10^3$. Using a commercially available CCD array with a frame rate of 1 MHz, a single array MQW encoding SLM with the frame rate of 1 MHz, 10×10 pixels MQW SLM for the coherence control with the frame rate of 10 MHz, and a commercial femtosecond fiber laser (PolarOnycs Inc., model Mercury) with pulse duration of 200 fs and repetition rate 30 MHz, we achieve the data rate of 2 Gigabits per second. Note, that although the narrowband assumption is valid in our case, $\Delta\lambda/\lambda = 0.0258 \ll 1$, for such short optical pulses the effects of the time spreading and fluctuation of arrival time due to the atmospheric turbulence become significant[29]. An increase of bit error rate appears when individual pulses overlap. Fortunately, in the spectral encoding method the time interval between pulses is much larger than the pulse duration. According to[29] the pulse spreading effect is described by $T = (T_0^2 + 8\alpha)^{1/2}$, where $\alpha = 0.391 C_0^2 L L_0^{5/3} c^{-2}$, C_0^2 is the refraction index structure constant, and L_0 is the outer-scale size of the turbulence. The fluctuation of the arrival time is $4\sigma^2 = T^2$. In our case the parameters are: $C_0^2 = 10^{-13} m^{-2/3}$, $L = 10^4 m$, $L_0 = 4\ m$, and $T_0 = 200 fs$. In this case, $T = 2.88 fs$. The time interval between pulses corresponding to the repetition rate, 30 MHz, is 33 ns. Thus, the time spreading and fluctuation of the arrival time are negligible compared with the interval between pulses.

6. CONCLUSION

We have presented the new concept of a free space, few Gbps speed optical communication system based on spectral encoding of radiation from a broadband pulsed laser. We have shown that, in combination with control of the partial coherence of the laser beam and a relatively slow photosensor, scintillations can be suppressed by orders of magnitude for communication distances beyond 10 km.

7. ACKNOWLEDGMENT

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