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METALS AND SUPERCONDUCTORS

Charge Density Waves in Partially Dielectrized *d*-Pairing Superconductors

A. I. Voĭtenko and A. M. Gabovich

Institute of Physics, National Academy of Sciences of Ukraine, pr. Nauki 46, Kiev, 680028 Ukraine
e-mail: collphen@iop.kiev.ua

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Abstract—A self-consistent theory has been constructed for describing a $d_{x^2-y^2}$ superconductor with a charge density wave caused by the appearance of a dielectric gap in antinodal sections of the two-dimensional Fermi surface. The theory explains some key features of high-temperature oxides. In particular, it has been shown that the observed large values of the ratio $2\Delta(T=0)/T_c$ are associated with the stronger suppression of the critical temperature T_c of the superconducting transition rather than the superconducting gap Δ at low temperatures T under the action of charge density waves. It has been predicted that there can exist two critical temperatures of the appearance and disappearance of the dielectric order parameter $\Sigma(T)$ in a specific range of bare parameters of the model.

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1. INTRODUCTION

The problem of coexistence of superconductivity and charge density waves has been intensively studied both experimentally and theoretically [1–9]. Nonetheless, a number of aspects of this problem remain unclear because of the complex many-body character of systems with the electronic spectrum reconstructed in the range of low temperatures T as a result of the combined action of the Cooper and electron–hole pairings. A wide spectrum of questions associated with the problem of combination of two order parameters in one object and in one temperature range became even more important when it was revealed that the presence of a pseudogap in cuprates, most probably, is also a manifestation of charge density waves both below and above the critical temperature T_c of a superconductor [4, 6, 7, 10–13]. However, there are opponents of this point of view despite the fact that, in essence, their understanding is based on the same experiments [14]. These alternative approaches make investigations in this direction very intriguing. The pseudogap can manifest itself in cuprates in different ways: as a decrease in the initial quasiparticle density of states below and above T_c, as a hump–dip structure observed in photoemission or tunneling spectra at low temperatures $T \ll T_c$, or as different superstructures with charge density modulation revealed using scanning tunneling microscopy [15], as well as X-ray [16, 17] or neutron spectroscopy [18].

It should be remembered that spectral features typical of the charge-density-wave phase in the superconductor can be smeared as a result of the spatial inhomogeneity, which can be inherent in high-temperature oxides and (or) can be caused by the specific features of the preparation of samples and oxygen distribution in the bulk and on the surface [4, 6, 19–22]. In this case, the spatial inhomogeneity associated with the oxygen distribution can manifest itself not only at the microlevel but also can lead to the phase separation [23, 24].

Both homogeneous and spatially inhomogeneous superconductors with charge density waves in the case of *s* symmetry of the superconducting order parameter were studied in detail in our earlier works [4, 6, 7, 25, 26]. Some of our results can be applied to cuprates only under certain reservations, because the superconducting order parameter Δ in these materials has been usually treated as having the $d_{x^2-y^2}$ symmetry [27, 28]. In the present work, we extended out previous theory to the case when superconductivity coexisting with charge density waves has an order parameter with the $d_{x^2-y^2}$ symmetry. On the basis of recent tunneling and photoemission experimental data, we constructed a model of a charge-density-wave superconductor and obtained a system of two equations for the order parameters $\Sigma(T)$ and $\Delta(T)$, which was solved numerically. The conclusions of this theory agree with the well-known and yet unexplained data on the ratio $\Delta(T=0)/T_c$, which is anomalously large for many cuprates (the Boltzmann constant is $k_B = 1$). We predicted new properties of the charge-density-wave superconductor; namely, in a certain range of model parameters, the quantity $\Sigma(T)$ has two critical temperatures; i.e., in a certain temperature range, there exists

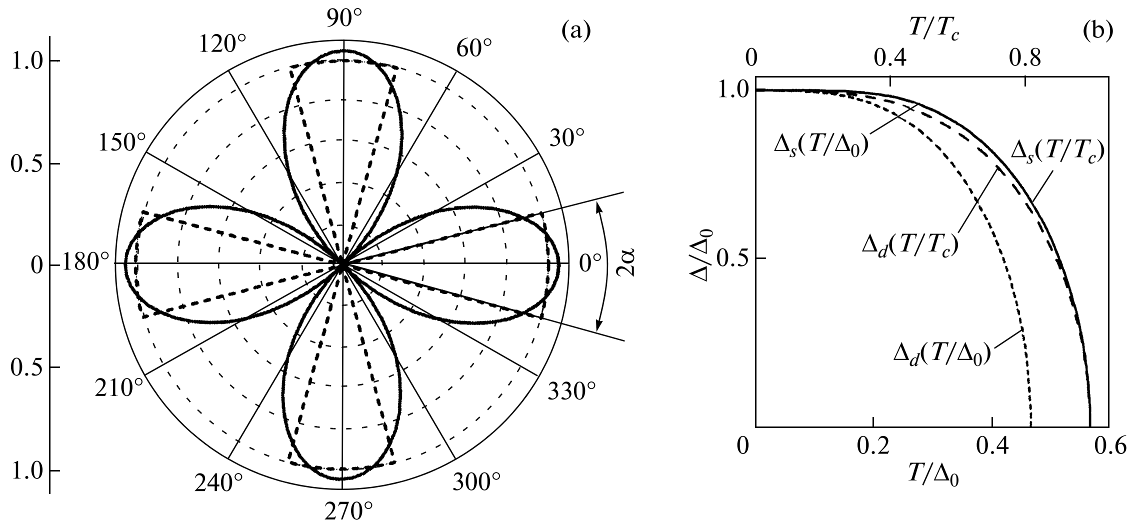

Fig. 1. (a) Profiles of the superconducting Δ (solid curve) and dielectric Σ (dashed curve) gaps in the two-dimensional momentum space for the parent phases of the $d_{x^2-y^2}$ superconductor and partially dielectrized metal with charge density waves, respectively; i.e., when the competitive pairing channel is switched off. (b) Normalized (in units of the gap value Δ_0 at zero temperature $T = 0$) dependences of the superconducting gaps with different symmetries on the temperature T normalized to Δ_0 and the corresponding critical temperature T_c for the parent phases.

a so-called reentrant behavior, which, hereafter, will be referred to as the reentrance.

2. THEORY

The theory suggested here is a nontrivial (from the physical and computational points of view) extension to the case of d pairing of the Bilbro–McMillan approach developed for the s (Bardeen–Cooper–Schrieffer) superconductivity [25, 29]. We perform calculations in the framework of a two-dimensional model for the Brillouin zone and Fermi surface. In this case, we ignore the quasiparticle dispersion along the c axis, which, in principle, should be taken into account in a more rigorous analysis [30]. The superconductivity is described in terms of the model with a weak electron–phonon coupling. The corresponding Hamiltonian is presented, e.g., in [31, 32]. According to photoemission [33–36] and scanning tunneling microscopy [15, 37–42] data, the charge-density-wave Hamiltonian considered below in the mean-field approximation is restricted to the momentum region near planar sections of the Fermi surface, which are antinodal from the viewpoint of four-lobe superconducting order parameter with the $d_{x^2-y^2}$ symmetry: $\Delta T(T)\cos^2 2\theta$ [27]. In these sections, the congruence conditions for the quasiparticle spectrum $\xi_{1i}(p) = -\xi_{2i}(p + Q_i)$ (where the index $i = 1, 2$ numbers the pair of opposite congruent sections of the Fermi surface) are fulfilled, so that there arise two charge density waves with the wave vectors Q_i (the Plank constant is $\hbar = 1$). For example, the scanning tunneling microscopy measurements evidence the existence of two wave vectors of electric-charge-density modulation in $Bi_2Sr_2CaCu_2O_{8+\delta}$ [15]; within 15%, it is $\mathbf{Q} = (\pm 2\pi/4.2a_0, 0)$ and $(0, \pm 2\pi/4.2a_0)$, where a_0 is the lattice constant in the CuO_2 plane. Moreover, the directions of these vectors coincide with those of the lobes of the d order parameter. Therefore, in accordance with the experimental data, we will consider the charge-density-wave modulation of the charge of the staggered type (symmetric with respect to the rotation by an angle multiple to $\pi/2$). Thus, we assume that both charge density waves are characterized by the same order parameter Σ.

Our d superconductor with a charge density wave is a phase with two interacting order parameters originating from the initial (bare) states. The first phase is a pure $d_{x^2-y^2}$ superconductor, whose properties are determined by the value of the superconducting order parameter Δ_0 at $T = 0$. In this state, i.e., in the absence of a charge-density-wave gap in the Fermi surface, the band gap would have the angular distribution in the momentum space as shown in Fig. 1a by the solid curve. At the same time, the dependence of the order parameter Δ on T would be described by the curve $\Delta_d(T/\Delta_0)$ in Fig. 1b. The second state is the aforementioned “pure” charge-density-wave metal with a partial dielectrization of the Fermi surface in four sectors centered around the directions coinciding with the maxima of the gap lobes $\Delta(T, \theta)$, each characterized by the opening angle 2α ($\alpha < \pi/4$). The dielectrization results from the electron–hole pairing of quasiparticles from the opposite congruent sections of the Fermi surface. Within each sector, the dielectric gap Σ is constant and equal to the magnitude of the dielectric order parameter; outside the sectors, the dielectric gap is absent (Fig. 1a, dashed curve). The temperature dependence $\Sigma(T)$ of the pure charge-density-wave metal coincides exactly with the Bardeen–Cooper–Schrieffer (BCS) dependence for the gap of an *s* superconductor (curve $\Delta_s(T/\Delta_0)$ in Fig. 1b) [43]. It is this circumstance that allows us to conventionally assign the *s* type to the dielectric order parameter. However, it should be noted that $\Sigma(T)$ is not isotropic in the momentum space (Fig. 1a).

The Dyson–Gor'kov equations for the normal and superconducting Green's function for a system with the Cooper *d* and electron–hole pairings are solved in a standard manner as in the case of an *s* superconductor with charge density waves [7, 25]. As a result, we obtain the system of coupled equations for the determination of $\Delta(T)$ and $\Sigma(T)$ (hereinafter, for brevity, we omit index *d* for quantities related to the *d* pairing in the cases where this does not lead to ambiguities); that is,

$$\int_0^{\mu\pi/4} I_M(\sqrt{\Sigma^2 + \Delta^2\cos^2 2\theta}, T, \Sigma_0)d\theta = 0, \tag{1}$$

$$\int_0^{\mu\pi/4} I_M(\sqrt{\Sigma^2 + \Delta^2\cos^2 2\theta}, T, \Delta_0)\cos^2 2\theta d\theta + \int_{\mu\pi/4}^{\pi/4} I_M(\Delta\cos 2\theta, T, \Delta_0)\cos^2 2\theta d\theta = 0. \tag{2}$$

Here, $\mu = 4\alpha/\pi$ is the dielectrization parameter that shows what part of the Fermi surface is dielectrized and varies in the range $0 < \mu < 1$; Δ_0 and Σ_0 are the bare values of the order parameters for the original *d* superconductor and charge-density-wave metal, respectively (each of them determines the value of the corresponding gap in the absence of a competitor); and

$$I_M(\Delta, T, \Delta_0) = \int_0^{\infty}\left(\frac{1}{\sqrt{\xi^2 + \Delta^2}}\tanh\frac{\sqrt{\xi^2 + \Delta^2}}{2T} - \frac{1}{\sqrt{\xi^2 + \Delta_0^2}}\right)d\xi \tag{3}$$

is the Muhlschlegel interval. The root $\Delta = sM\ddot{u}(\Delta_0, T)$ of Eq. (3) is the well-known Bardeen–Cooper–Schrieffer dependence for the gap of the *s* superconductor (curve $\Delta_s(T/T_c)$ in Fig. 1b). It should be emphasized once again that the domain of existence of the *d* gap Δ is the entire Fermi surface, whereas the gap Σ exists only in the degenerate congruent sections of the Fermi surface that are bounded by cones with the apex angle 2α defined above (Fig. 1a).

Naturally, the system of equations (1) and (2) describes not only the state of the d superconductor with charge density waves ($\Delta \neq 0$, $\Sigma \neq 0$) but also the cases when one of the order parameters is zero. In the absence of superconductivity, e.g., above T_c but below the charge-density-wave transition temperature T_{CDW0}, Eq. (1) becomes formally independent of μ and $\Sigma(T) = sM\ddot{u}(\Sigma_0, T)$ with $\Sigma_0 = \frac{\pi}{\gamma}T_{CDW0} = 2W\exp[-1/V_{CDW}\mu N(0)]$, where $\gamma = 1.78...$ is the Euler constant, V_{CDW} is the matrix element of the electron–hole interaction, W is the cutoff energy of electron–hole pairing, and N_0 is the total density of states on the Fermi surface in the normal phase. Thus, in this case, the system represents the pure original charge-density-wave phase.

On the other hand, in the absence of charge density waves, Eq. (2) transforms into the relationship

$$\int_0^{\pi/4} I_M(\Delta\cos 2\theta, T, \Delta_0)\cos^2 2\theta d\theta = 0 \tag{4}$$

and describes a conventional *d*-pairing superconductor. The solution to this equation $\Delta = dM\ddot{u}(\Delta_0, T)$ is also well known (curve $\Delta_d(T/T_c)$ in Fig. 1b) [31, 32]. In particular, the critical temperature is $T_{c0} = \frac{2\Omega\gamma}{\pi}\exp[-1/V_{BCS}N(0)]$, where Ω and V_{BCS} are the cutoff energy and the matrix element for the Cooper pairing, respectively. Equation (4) leads to a change in the ratio between the gap and the critical temperature as compared to the case of *s* pairing,

$$\left(\frac{\Delta_0}{T_{c0}}\right)_d = \frac{2}{\sqrt{e}}\frac{\pi}{\gamma} = \frac{2}{\sqrt{e}}\left(\frac{\Delta_0}{T_{c0}}\right)_s, \tag{5}$$

which agrees with the results obtained in [31] (*e* is the base of natural logarithms). Thus, Eq. (4) plays the same role for the *d* superconductor as Eq. (3) for its *s* counterpart.

Strictly speaking, the system of equations (1) and (2) for the *d* superconductor with charge density waves can be easily transformed into the corresponding system for the *s* superconductor with charge density waves by simply changing $\cos^2 2\theta$ by unity. The same is true for Eqs. (4) and (3). Nonetheless, the presence of the factor $\cos^2 2\theta$ in one of the two coupled equations for the order parameters is crucial and determines the properties of the *d* superconductor with charge density waves. This circumstance is associated with the fact that the symmetries of both parameters are different. This physical reason leads to interesting mathematical consequences.

Indeed, it was shown in our previous work [25] that, in the case of *s* superconductors with charge density waves, the quantities $\Delta_s(T)$ and $\Sigma_s(T)$ in the entire range where $\Delta \neq 0$ and $\Sigma \neq 0$ are related by the simple expression $\Delta_s^2(T) + \Sigma_s^2(T) = [sM\ddot{u}(\Sigma_0, T)]^2$; i.e., apart from the temperature T, the determining parameter is Σ_0 and the parameters Δ_0 and μ do not enter into this expression. Moreover, both order parameters retain nonzero values with a decrease in T to $T = 0$ [25]. The coexistence of the charge density waves and the isotropic superconductivity is possible only when the ine-

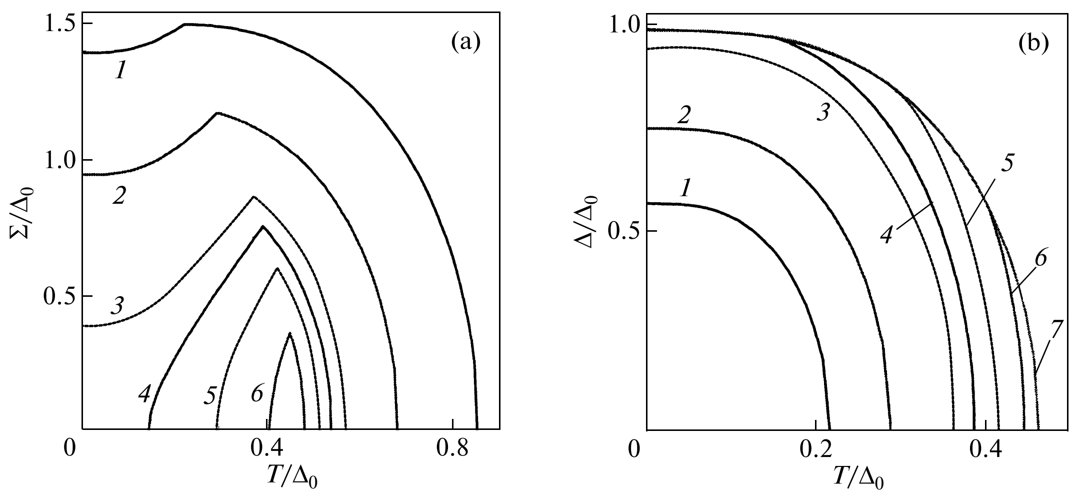

Fig. 2. Temperature dependences of (a) the dielectric Σ and (b) superconducting Δ order parameters for the $d_{x^2-y^2}$ superconductor with charge density waves for different ratios of the bare gap parameters Σ_0/Δ_0 = (*1*) 1.5, (*2*) 1.2, (*3*) 1, (*4*) 0.95, (*5*) 0.9, (*6*) 0.85, and (*7*) 0.8. The degree of dielectrization of the Fermi surface is $\mu = 0.3$.

quality $\Delta_0 < \Sigma_0$ holds true. Otherwise, taking into account the same functional dependences $\Sigma_s(T)$ and $\Delta_s(T)$ for the same type (s) of pairing, the inequality $\Sigma_s(T) < \Delta_s(T)$ would be fulfilled for any T and the dielectric gap could not appear on the Fermi surface uniformly "covered" by the superconducting gap (at the same time, the appearance of the superconducting s gap on a partially dielectrized Fermi surface is always possible [25, 29, 44, 45]). In the case of d superconductors with charge density waves, the situation changes radically. The superconducting $d_{x^2-y^2}$ gap varies in the momentum space from the maximum value to zero, and the domain of system parameters in which the coexistence of the charge density waves and the d superconductivity is possible is extended substantially. It can be seen from Eq. (1) that the relationship between $\Delta(T)$ and $\Sigma(T)$ becomes more complex than that for isotropic superconductors and now depends not only on Σ_0 and T but also on μ. The decisive factor is that, at the same critical temperatures $T_c = T_{CDW0}$, the temperature dependence of the d gap Δ is steeper than the dependence $\Sigma(T)$. Moreover, the corresponding zero value Δ_0 exceeds Σ_0. However, this small quantitative difference leads to noticeable qualitative consequences.

3. NUMERICAL RESULTS AND THEIR ANALYSIS

It can be seen from Fig. 2a that a decrease in the bare charge-density-wave gap Σ_0 when the parameters Δ_0 and μ remain constant leads to the transformation of $\Sigma(T)$ from the form with a sharp kink at $T = T_c$ and concave portion for $T < T_c$ (this shape is similar to that for s superconductors with charge density waves, which takes place for any admissible bare parameters [25]) into the form characterizing a radically new reentrant behavior of the dielectric order parameter. Figure 2b shows the corresponding dependences $\Delta(T)$. For all relationships between the parameters of the problem, they retain their monotonic behavior.

Let us consider the interrelation between two order parameters in more detail (the dependences shown in Fig. 3 correspond to curves *4* in Fig. 2). From the physical point of view, the superconductivity (no matter, s or d) and charge density waves are antagonists,

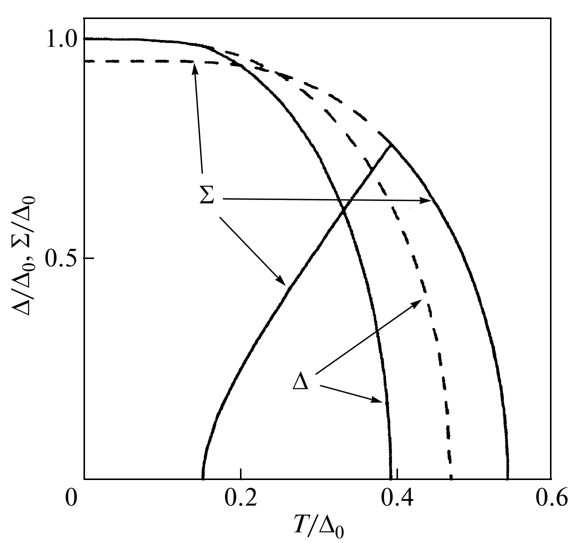

Fig. 3. Comparison of the temperature dependences of the order parameters Δ and Σ for the parent phases (dashed curves) and the phases with competitive ($d_{x^2-y^2}$) Cooper and electron–hole pairings (solid curves) for $\Sigma_0/\Delta_0 = 0.95$ and $\mu = 0.3$.

because the pairing interactions of quasiparticles both in the Cooper and electron–hole channels tend to reduce the energy of the electronic subsystem by forming the corresponding gaps on the same Fermi surface [7, 46]. At $T_{\rm CDW0} > T_{c0}$, with a decrease in T, the dielectric order parameter Σ appears first and varies in the same manner as it should be in the pure charge-density-wave phase. Since the arising charge density wave tends to suppress the superconductivity and the parameter Σ itself very rapidly ($\propto\sqrt{T_{\rm CDW0}-T}$) increases near $T_{\rm CDW0}$, as T approaches T_{c0}, it already is sufficiently large to suppress the origination of the superconductivity in a certain range below T_{c0}, shifting its appearance to a certain real critical temperature T_c. In this case, the bare dependence $\Delta = dM\ddot{u}\,(\Delta_0, T)$ as if contracts on the side of high temperatures and becomes even steeper (see the dashed and solid dependences Δ in Fig. 3). Recall that the appearance of the superconducting order parameter is inevitable because the dielectrization of the Fermi surface is only partial. At the same time, the superconducting order parameter propagates over the entire Fermi surface, including its dielectrized sections due to the strong mixing of states from different sections of the Fermi surface [29]. In this case, there arises a "back" suppressive action of Δ on Σ. Here, the possible scenarios depend on the steepness and amplitude of the gap function $\Delta(T)$. If T_c is sufficiently far from $T_{\rm CDW0}$, the "power" of the superconducting order parameter, which is already suppressed by the charge density waves, at $T < T_c$, is sufficient only to deviate $\Sigma(T_c)$ from the bare dependence $sM\ddot{u}\,(\Sigma_0, T)$. In this case, we should observe the dependences $\Delta(T)$ and $\Sigma(T)$ (Fig. 2, curves *1–3*) similar to those characteristic of the *s* superconductor with charge density waves. If the model parameters are such that the "residual power" of the superconducting order parameter continues to increase sufficiently rapidly with a decrease in T, the order parameter Σ can be completely suppressed at low T (Fig. 2a, curves *4–6*) and the superconducting order parameter in this range of T will be determined by the bare dependence $\Delta = dM\ddot{u}\,(\Delta_0, T)$ (Fig. 2b, curves *4–6*). It is significant that the reentrance phenomenon is reproduced in the framework of an extremely simple model with two competitive order parameters having different dependences on the transferred momentum. It should be noted that the charge-density-wave structures in real systems can be considerably more complex with nonmonotonic dependences on the temperature T even in the absence of superconductivity [11].

Let us formulate the conditions necessary for appearing the reentrance phenomenon in the objects under study. Reasoning from the aforementioned mechanism of interaction between both order parameters and relationship (5), in order that the charge-density-wave phase could arise at temperatures below some $T^{u}_{\rm CDW}$, it is necessary that the inequality $T^{u}_{\rm CDW} = \frac{\gamma}{\pi}\Sigma_0 > T_{c0} = \frac{\sqrt{e}\gamma}{2\pi}\Delta_0$ should be satisfied. In other words, the condition $\Sigma_0 > \frac{\sqrt{e}}{2}\Delta_0 \approx 0.824\Delta_0$ should hold true (compare it with the condition $\Sigma_0 > \Delta_0$ for the superconductor with charge density waves). It should be noted that considerations regarding the coexistence of the charge density waves and the superconductivity were not used in this case, so that the derived inequality does not involve the controlling parameter μ. It is evident that the quantity $T^{u}_{\rm CDW}$ thus obtained coincides with $T_{\rm CDW0}$.

On the other hand, if there exists a lower nonzero critical temperature $T^{l}_{\rm CDW}$ for the domain of existence of the dielectric order parameter; i.e., if the domain of nonzero values of Σ is bounded by the temperature range $T^{l}_{\rm CDW} < T < T^{u}_{\rm CDW}$, we have $\Sigma(T) = 0$ and $\Delta(T) = dM\ddot{u}\,(\Delta_0, T)$ at $T \le T^{l}_{\rm CDW}$. Since both order parameters vary continuously, in order to determine $T^{l}_{\rm CDW}(\Delta_0, \Sigma_0, \mu)$, we should use Eq. (1) after the substitutions $T = T^{l}_{\rm CDW}$ and $\Delta(T^{l}_{\rm CDW}) = dM\ddot{u}\,(\Delta_0, T^{l}_{\rm CDW})$. The derived equation can be solved only numerically. However, the characteristic value of the parameter $\Sigma_0^{\rm cr}$ corresponding to the separatrix that divides the dependences $\Sigma(T)$ into two types, i.e., with reentrance and without reentrance, can be obtained in an analytical form, because this corresponds to the case when $T^{l}_{\rm CDW} = 0$. Then, by using elementary calculations, from Eq. (1), we find

$$\Sigma_0^{\rm cr} = \Delta_0 \exp\left[\frac{4}{\mu\pi}\int_0^{\mu\pi/4}\ln(\cos 2\theta)\,d\theta\right]. \qquad (6)$$

In particular, curves in Fig. 2 were calculated for $\mu = 0.3$, so that the scenario with reentrance of Σ occurs in the following range of the bare intensity of electron–hole pairing: $0.824\Delta_0 < \Sigma_0 < 0.963\Delta_0$. This result agrees with the numerical solutions. It should be emphasized once again that the domain of coexistence of both order parameters for *d* superconductors with charge density waves is wider (the coexistence is possible for the relationships $\Sigma_0/\Delta_0 \gtrless 1$ under certain constraints on μ) than that for *s* superconductors with charge density waves, for which the stringent condition $\Sigma_0/\Delta_0 > 1$ should be satisfied [25].

It follows from the aforesaid that, depending on the relationships between the bare parameters Δ_0, Σ_0, and μ, the charge density waves in *d* superconductors can manifest themselves in different manners; namely, the charge density waves are most probably responsible for the pseudogap decrease in the density of states above

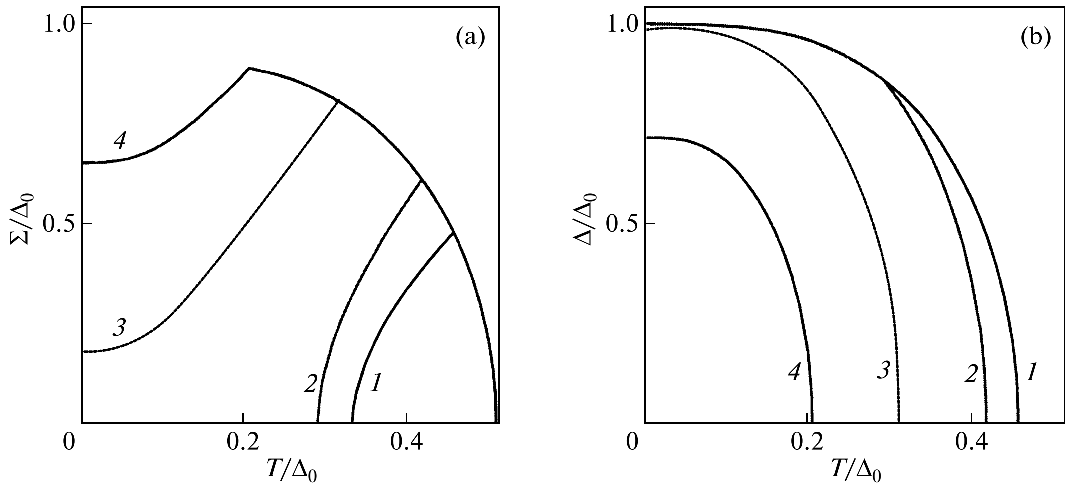

Fig. 4. The same as in Fig. 2 but for $\Sigma_0/\Delta_0 = 0.9$ and $\mu =$ (*1*) 0.1, (*2*) 0.3, (*3*) 0.5, and (*4*) 0.6.

T_c and below T_c [4, 6, 47]. The hump–dip structure at low T in tunneling, microcontact, and photoemission spectra according to our concept can be either observed or not observed (the experimental data are presented in reviews [48, 49]) depending on whether the reentrance phenomenon for Σ takes place. The consequences from the proposed theory can be experimentally verified by varying the relationships between the controlling parameters. For example, the uniform compression makes it possible to vary the degree of dielectrization of the Fermi surface μ in certain limits. All the parameters of the superconductor with charge density waves can be varied by doping it, e.g., with oxygen.

Figure 4 shows the dependences $\Sigma(T)$ and $\Delta(T)$ for $\Delta_0 = 1$, $\Sigma_0 = 0.9$, and different μ. It can be easy to see that the suppression of the dielectric gap Σ by the superconductivity at low T is strong if the degree of dielectrization of the Fermi surface is low. This situation can be observed, for example, in $Bi_2Sr_2CaCu_2O_{8+\delta}$ [50] and $(Bi,Pb)_2(Sr,La)_2CuO_{6+\delta}$ [51], in which the doping with oxygen leads to a drastic decrease in the parameter μ. It is also worth noting that the dependences $\Delta(T)$ are distorted by the action of charge density waves and do not coincide with the normalized canonical curve $dMü(T)$ in contrast to the case of the s superconductor with charge density waves [25]. Therefore, different shapes of the dependence $\Delta(T)$ observed experimentally cannot uniquely indicate the true symmetry of superconducting pairing. Moreover, the problem of symmetry of the order parameters in cuprates is far from the final solution [52]; in particular, the phase with the order parameter having s and d components can occur in these compounds [53].

It is obvious that the influence of charge density waves on the d superconductivity should change the ratio $2\Delta(0)/T_c$, which is a characteristic quantity indicating the strength of the superconducting pairing (for a weak coupling, expression (5) is valid). The modification of $2\Delta(0)/T_c$ under the action of charge density waves can be easily understood if we remember how the dependence $\Delta(T)$ contracts on the side of high temperatures (Fig. 3). At the same time, this ratio in s superconductors with charge density waves remains identical to that in ordinary Bardeen–Cooper–Schrieffer superconductors [25]. Figure 5a shows the dependences of the ratios $2\Delta(0)/T_c$ and T_c/Δ_0 on Σ_0. The value of $2\Delta(0)/T_c$ sharply increases with increasing Σ_0 for $\Sigma_0 \leq 1$ and tends to saturation for large Σ_0, whereas T_c/Δ_0 decreases almost uniformly. In particular, at $\mu = 0.3$, the saturation level of $2\Delta(0)/T_c$ is equal to 5.2. Such a significant increase is in good agreement with the experimental data for cuprates [54–56]. It should be emphasized that this good agreement cannot be achieved in the framework of the strong electron–boson interaction for reasonable ratios between T_c and the effective boson frequency ω_E [57, 58] (it is unlikely that the value of $T_c/\omega_E \approx 0.3$ [57] can be considered justified from the practical point of view).

The dependences of the same ratios on μ are plotted in Fig. 5b. It is readily seen that $2\Delta(0)/T_c$ can reach rather large values if the degree of dielectrization of the Fermi surface is sufficiently high. However, this increase takes place with a significant decrease in T_c, which eventually leads to a fast disappearance of the superconductivity. Most likely, this situation takes place in underdoped cuprates, when the decrease in T_c is accompanied by a considerable increase in the superconducting gap. For example, this phenomenon was observed in junctions for $Bi_2Sr_2CaCu_2O_{8+\delta}$ samples over a wide range of oxygen concentrations [59].

In [54], it was noted that photoemission and tunneling experiments for different families of high-temperature oxides lead to a characteristic ratio

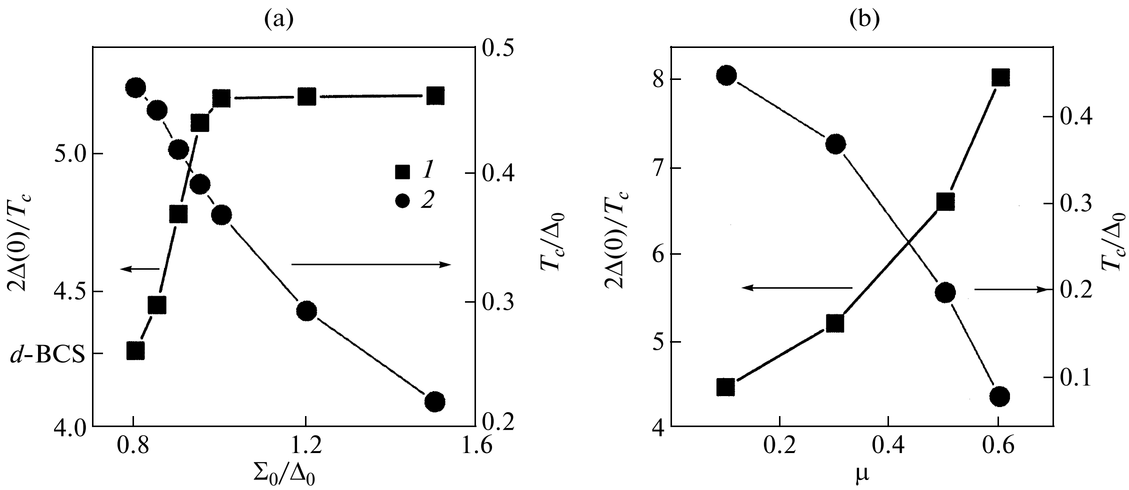

Fig. 5. Dependences of (*1*) $2\Delta(0)/T_c$ and (*2*) T_c/Δ_0 on (a) Σ_0/Δ_0 ($\Delta_0 = 1$, $\mu = 0.3$) and (b) μ ($\Delta_0 = 1$, $\Sigma_0 = 1$). T_c is the critical temperature of the charge-density-wave phase of the *d* superconductor. The value of $2\Delta(0)/T_c \approx 4.28$ for the "pure" $d_{x^2-y^2}$ Bardeen–Cooper–Schrieffer superconductor (*d*-BCS) is shown.

$2\Delta(0)/T_c \approx 5.5$. According to Fig. 5b, this ratio corresponds to $\mu \approx 0.35$ for $\Sigma_0 = 1$. Then, the other curve in the same figure gives $T_c/\Delta_0 \approx 0.35$. Since $(\Delta_0/T_{c0})_d \approx 2.14$ (see relationship (5)), we obtain $T_c/T_{c0} \approx 0.75$. This is a rather reasonable evaluation for the influence of charge density waves that leads to a decrease in T_c.

4. CONCLUSIONS

Thus, in the present work, we constructed the theory of coexistence between the $d_{x^2-y^2}$ superconductivity and charge density waves, which follows from the set of the available experimental data. It was shown that the interrelation between the corresponding order parameters Δ and Σ differs from that for the case of *s* superconductors. In particular, for specific parameters of charge density waves in our problem, the order parameter $\Sigma(T)$ exists in the temperature range bounded both from above and below. On the other hand, the resulting dependence $\Delta(T)$ is also strongly distorted under the action of charge density waves, because the ratio $2\Delta(T = 0)/T_c$ can take on large values substantially exceeding the Bardeen–Cooper–Schrieffer value for *d* superconductors. It seems likely that this can explain the experimental values of $2\Delta(0)/T_c \approx$ 5–8 obtained for high-temperature oxides [54–56].

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